AD-A034 460 PENNSYLVANIA STATE UNIV UNIVERSITY PARK APPLIED RESE--ETC F/G 13/9
MECHANICAL ANTIVIBRATION FILTER DESIGN USING ELECTRICAL NETWORK--ETC(U)
APR 76 C B PUTMAN N00017-73-C-1418
UNCLASSIFIED TM-76-162 NL

1 OF 2
ADA034460

FG.

12

MECHANICAL ANTIVIBRATION FILTER DESIGN
USING ELECTRICAL NETWORK SYNTHESIS TECHNIQUES

Craig B. Putnam

Technical Memorandum
File No. TM 76-162
April 7, 1976
Contract No. N00017-73-C-1418

Copy No. 7

The Pennsylvania State University
Institute for Science and Engineering
APPLIED RESEARCH LABORATORY
Post Office Box 30
State College, PA 16801

NAVY DEPARTMENT

NAVAL SEA SYSTEMS COMMAND

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER TM-76-162	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MECHANICAL ANTIVIBRATION FILTER DESIGN USING ELECTRICAL NETWORK SYNTHESIS TECHNIQUES		5. TYPE OF REPORT & PERIOD COVERED M.S. Thesis, November 1976
		6. PERFORMING ORG. REPORT NUMBER TM 76-162
7. AUTHOR(s) Craig B. Putnam		8. CONTRACT OR GRANT NUMBER(s) N00017-73-C-1418
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Pennsylvania State University Applied Research Laboratory P. O. Box 30, State College, PA 16801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Sea Systems Command Department of the Navy Washington, D.C. 20362		12. REPORT DATE April 7, 1976
		13. NUMBER OF PAGES 99 pages & figures
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Master's thesis		15. SECURITY CLASS. (of this report) Unclassified, Unlimited
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

14 6 10 15 11 9 12 100p.

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited, per NSSC (Naval Sea Systems Command), 6/23/76

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Mechanical filters
Fiber Network Design
Vibration Isolators
Antivibration filters

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Two methods are presented by which mechanical low-pass filters can be designed using electrical network synthesis techniques. The desired response can be any low-pass filter function that is realizable as a planar reactive ladder network. The resultant response of the synthesized network will be slightly different from the desired response, but this difference is predictable and can be taken into account when considering the specifications for the design of the filter.

These techniques can be used to design either a filter network with a relatively small source mass driven by a constant force or a filter network with a

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

391 00

20. ABSTRACT (Continued)

relatively large source mass moving with a constant velocity. The two methods are compared and an example is given to illustrate the implementation of one of the methods.

ABSTRACT

Two methods are presented by which mechanical low-pass filters can be designed using electrical network synthesis techniques. The desired response can be any low-pass filter function that is realizable as a planar reactive ladder network. The resultant response of the synthesized network will be slightly different from the desired response, but this difference is predictable and can be taken into account when considering the specifications for the design of the filter.

These techniques can be used to design either a filter network with a relatively small source mass driven by a constant force or a filter network with a relatively large source mass moving with a constant velocity. The two methods are compared and an example is given to illustrate the implementation of one of the methods.

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation for the guidance and encouragement rendered by his advisor, Richard O. Rowlands, and by the other members of his committee. The author also acknowledges the support of the Applied Research Laboratory of The Pennsylvania State University under contract with the Naval Sea Systems Command, and the use of its facilities in the preparation of this thesis.

TABLE OF CONTENTS

		Page
ABSTRACT		ii
ACKNOWLEDGEMENTS		iii
LIST OF SYMBOLS		vi
LIST OF FIGURES		viii
I.	INTRODUCTION	
1.1	Statement of the Problem	1
1.2	Purpose of the Research	3
1.3	Analysis Techniques	4
II.	GENERAL NETWORK SYNTHESIS TECHNIQUES	
2.1	Network Selection Criteria	5
2.2	Synthesis Methods Suitable to This Problem	10
2.3	Darlington Synthesis Method	11
2.4	Cauer and Guillemin Synthesis Method	16
III.	METHODS YIELDING LOW MASS SOURCES	
3.1	Modified Transfer Admittance Method (Darlington)	20
3.2	Modified Transfer Admittance Method (Cauer and Guillemin)	27
3.3	Response Characteristics	30
3.4	Network Limitations	34
IV.	METHODS YIELDING HIGH MASS SOURCES	
4.1	Capacitor Shift Method	38
4.2	Capacitor Shift Method (Cauer and Guillemin)	43
4.3	Response Characteristics	45
4.4	Network Limitations	58
V.	EXTENSION OF THE SYNTHESIS METHODS TO THE GENERAL CASE	
5.1	General Limitations	59
5.2	Derivation of the Output Modifying Function (M.T.A.)	59
5.3	Derivation of the Final Capacitor Lower Limit (M.T.A.)	63
5.4	Derivation of the Output Modifying Function (C.S.)	66
5.5	Derivation of the Final Capacitor Lower Limit (C.S.)	67

TABLE OF CONTENTS (cont.)

	Page
VI. THEORETICAL EXAMPLE	
6.1 A Low Mass Source Force Generator	70
VII. CONCLUSION AND RECOMMENDATION FOR FURTHER RESEARCH	80
BIBLIOGRAPHY	90

LIST OF SYMBOLS

$Z^T(s)$	Transfer impedance
$Y^T(s)$	Transfer admittance (Note: Admittance notation same as impedance)
$Z_{11}(s)$	Input impedance with output open circuited
$Z_{12}(s)$	Mutual impedance with input and output open circuited
$Z_{22}(s)$	Output impedance with input open circuited
$Z_n(s)$	Impedance of the n-th element
$Z_{22}^n(s)$	Impedance looking into the output after n elements are removed
α	Scale factor used in adjusting element values
s	Complex frequency (radians/sec)
$[Z_{in}(s)]_{MR}$	Minimum reactive input impedance
$Z_M^T(s)$	Modified transfer impedance
V_1,V_s	Input voltage
V_2,V_3	Output voltage
I_1,I_{in}	Input current
I_2,I_3	Output current
M	Mass
S	Spring
R	Resistance
L	Inductance
C	Capacitance
a	Value of the inserted capacitance

LIST OF SYMBOLS (cont.)

b	Value of the normalized inserted capacitance
H	Constant determining gain of network
K, ξ	Constants
ω	Frequency (radians/sec)
P	Power
$Re(\ )$	Real part of ( )
R_{in}	Real part of input impedance
F_n	Force on the n-th element of the mechanical network

LIST OF FIGURES

Figure		Page
1.1	Simple Filter Systems	5
1.2	General Response of a Two-Degree-of-Freedom System . . .	6
2.1(a)	Typical Mechanical Low-Pass Filter	8
2.1(b)	Equivalent Electrical Low-Pass Filter	9
2.2	Darlington Network	12
2.3	Ladder Network	17
2.4	Partial Synthesis of Reactive Ladder Network	20
3.1	3-Pole Butterworth Network	22
3.2	Desired Modified 3-Pole Butterworth Network	22
3.3	Network Synthesizing $\lvert Y_M^T(s)\rvert^2 = \dfrac{-a^2s^2}{1 - a^2s^2 - s^6 + a^2s^8}$.	26
3.4	Cauer and Guillemin Synthesis of $Y_M^T(s)$	29
3.5	Analagous 3-Pole Butterworth Networks (Darlington) . . .	32
3.6	$\lvert V_2/V_1\rvert^2$ vs. Omega for the Modified Transfer Admittance Method	33
4.1	Equivalent 3-Pole Butterworth Networks	39
4.2	Impedance Transformation Circuits	41
4.3	Capacitor Shift Circuit Transformations	42
4.4	Analagous 3-Pole Butterworth Networks (Capacitor-Shift) .	46
4.5	Response $\lvert V_2/I\rvert^2$ of Network in Figure 4.4(a)	50
4.6	Equivalent Source Configurations	52
4.7	Force Driven Network	53

LIST OF FIGURES (cont.)

Figure		Page
4.8	$\|V_2/V_s\|^2$ for Network in Figure 4.7	55
4.9	Raising Cutoff Frequency to Meet Response Criteria . . .	57
5.1	Output Modifying Function	61
5.2	Modified Transfer Admittance Method on a General Impedance	64
5.3	Shifting a Capacitor Through a General Network	68
6.1	Analagous 4-Pole Butterworth Networks with a Zero at Twice the Cutoff Frequency	72
6.2	Table of Element Values for Figure 6.1(b) as a Function of 'a'.	74
6.3	Output Modifying Function for 4-Pole Butterworth Network	76
6.4	$\|V_2/V_1\|^2$ for the Network in Figure 6.1(a)	77
6.5	Equivalent Damped Resonant Circuits	80
6.6	$\|V_2/V_1\|^2$ for the Network in Figure 6.1(a) with Damping in the Resonant Arm	81
6.7	Detail of Response Near Resonance	85
6.8	$\|V_2/V_1\|^2$ for the Network in Figure 6.1(a) Showing 12 dB Per Octave Rolloff Above Resonance	86

CHAPTER I

INTRODUCTION

1.1 Statement of the Problem

In designing an antivibration filter or vibration isolator, the designer strives to have the filter meet certain criteria. Usually, these criteria are that the filter has (1) a low resonant frequency, (2) a reasonably damped resonant peak, and (3) an adequate amount of rolloff above the resonance.

The first and second criteria are usually met by adjusting the mass of the source or the stiffness constant of the filter and by adjusting the damping of the filter at resonance. Meeting the third criterion, however, may not be so easy.

Whenever a filter needs more than a 12 dB per octave rolloff above resonance, the simple spring-mass resonator shown in Figure 1.1(a) will not suffice. A compound or multi-degree-of-freedom system as shown in Figure 1.1(b) can give any multiple of 12 dB per octave rolloff depending on the number of stages used. However, there will in general be N resonance peaks corresponding to the N stages. The response curve showing force transmissibility ($|FORCE_{output}/FORCE_{input}|^2$) will look something like the curve shown in Figure 1.2 for $N = 2$. Figure 1.2 is typical of the transmissibility curves for a two-degree-of-freedom system as treated by Snowdon.[(1)] By adjusting the damping in each

stage, the peaks can be reduced or even removed, but this is a rather inefficient way to go about producing a particular rolloff function.

It has long been known that there exist basic analogies between electrical and lumped component mechanical networks, such as the type of networks shown in Figures 1.1(a)-(c), where the springs are considered massless and perfect. These analogies make it possible to express the behaviour of a mechanical network in terms of the behaviour of its analogous electrical network.

Approximately thirty years ago, methods were developed by which two terminal pair electrical networks having specified realizable transfer functions could be synthesized. Most of these networks are filters consisting of purely reactive components terminated at one or both ends in resistors. The advantage of this in the design of mechanical networks is that the terminating resistance provides the damping for the entire network. In the case of a "maximally-flat" transfer function, the terminating resistance provides critical damping for the network and thereby eliminates any resonant peaks.

Mechanical antivibration filters are analogous to electrical low-pass filters and should therefore be subject to the same methods of design. In this way, a mechanical network can be produced having any realizable transfer function. There are, however, some practical requirements to be taken into account. First, the electrical network that produces the desired transfer function may not result in a desirable mechanical network due to the presence of transformers or to a mechanically undesirable topology. Second, the electrical synthesis methods produce networks that are driven by ideal voltage or current

generators which correspond to ideal force or velocity sources. Any real source will, of course, not be ideal and this will produce some deviation away from the desired response. Third, if the filtering network is to support the vibrating object, the final supporting member must be a spring since a mass cannot be supported on a dashpot. The termination of the analogous electrical network will therefore be a capacitor in series with a resistor, that is, a load containing reactive components.

Since most of the synthesis methods of practical interest result in networks terminated in an open circuit, a short circuit, or a purely resistive load, the required series complex load will undoubtedly affect the performance of the network. This paper will deal with the problem of modifying the classical synthesis techniques to produce the required complex load and how these modifications will affect the desired network response.

1.2 Purpose of the Research

While research in antivibration filter design has been going on for many years, most of the designs fall into one of three categories. Figures 1.1(a) and (b) denote the first two categories and Figure 1.1(c) is representative of the third category - the dynamic absorber. All three of these categories and their many permutations have been treated extensively by Snowdon[2] for rubber or rubberlike mounts. There does not appear to have been developed a general theory that can be applied to the design of an antivibration filter of any required degree of complexity. The purpose of this research is to attempt to develop such

a theory. In this study, it will be assumed in the beginning that all the elements are simple, i.e., as being lumped and having a mass, compliance or resistance that is independent of frequency. The effect of viscous damping in the springs will be discussed afterward.

1.3 Analysis Techniques

Classically, the response to excitation of a spring-mass system has been analyzed by the use of differential equations. The requirement of a complex response function, however, would result in a very difficult equation to solve. It was for this reason that a different method of analysis has been developed. The method adopted is one that has been used in the classical development of filter theory, namely, to calculate the impedance or admittance as a function of the complex frequency 's'. The transfer impedance and admittance are defined as

$$Z^T(s) = V_2/I_1 \quad \text{and} \quad Y^T(s) = I_2/V_1 \quad \text{, respectively.}$$

There are two ways to represent the mechanical analog of an electrical network, namely the "impedance" and "mobility" analogs. For the sake of consistency, it was necessary to make a choice between the two representations. The representation chosen for use throughout this paper is the "impedance" analogy. In this analogy, a mass is represented as an inductor, a spring as a capacitor and damping as resistance. Also, force is analagous to voltage and velocity to current. Therefore, a series constant voltage generator will represent a constant force source and a parallel constant current generator will represent a constant velocity source.

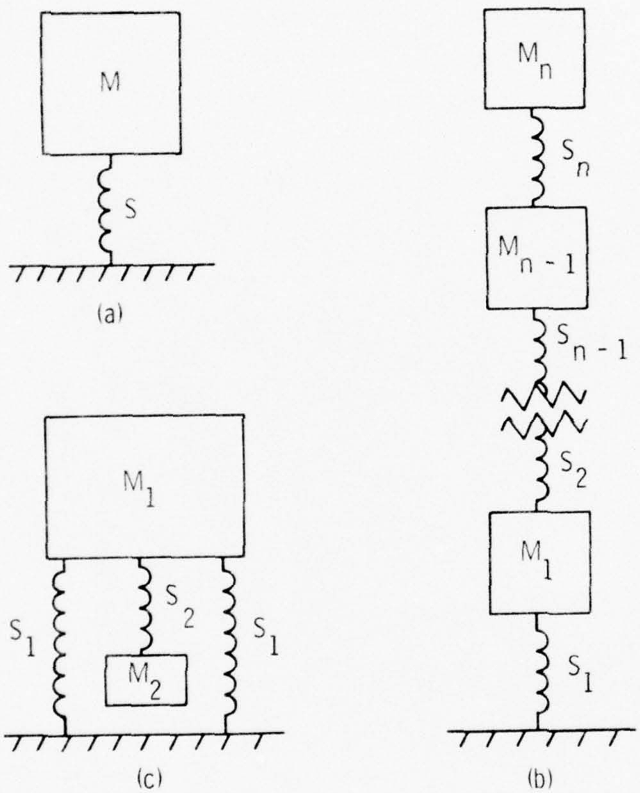


Figure 1.1 Simple Filter Systems

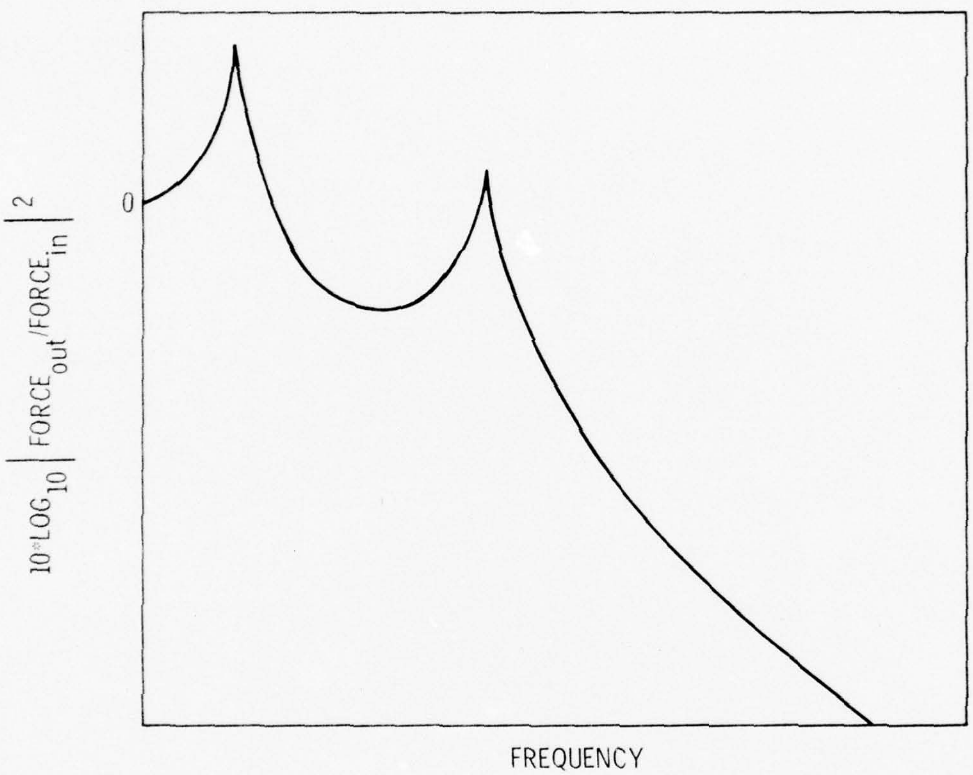


Figure 1.2 General Response of a Two-Degree-of-Freedom System

CHAPTER II

GENERAL NETWORK SYNTHESIS TECHNIQUES

2.1 Network Selection Criteria

In designing a low-pass mechanical filter capable of supporting a vibrating object, there are several criteria to be considered.

1. The filter should be simple in design. Figure 2.1(a) shows the type of mechanical network under consideration to control vibration in the vertical plane. There may be resonances in the system which would be achieved by the spring-mass resonator consisting of elements S_2 and M_2 . Also, the element above the supporting spring need not be a mass; it could be another spring. Note that the electrical equivalent of Figure 2.1(a) is the reactive ladder network shown in Figure 2.1(b). Also note that the last supporting member in Figure 2.1 (a) is, as required, a spring designated 'S' in parallel with a dashpot designated 'R'.

2. Space and cost considerations require the filter to be as compact as possible. Since these factors will be proportional to the number of elements, synthesis methods yielding canonic or nearly canonic networks are desirable.

3. Because of the variety of situations in which a low-pass mechanical filter is used, there will be many different response functions required. The synthesis method must be able to yield a network having any desired realizable low-pass response function.

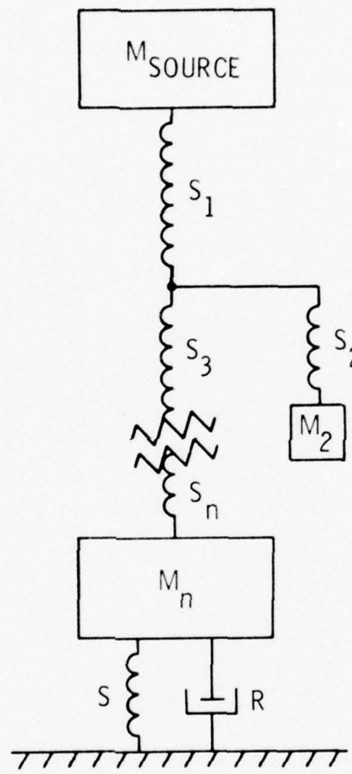


Figure 2.1(a) Typical Mechanical Low-Pass Filter

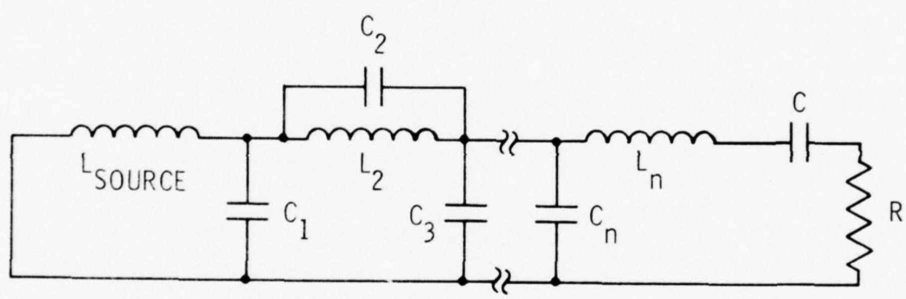


Figure 2.1(b) Equivalent Electrical Low-Pass Filter

4. In an antivibration filter, the stop band is of greater interest than the pas band. Any undesirable characteristics of the filter, such as resonances or deviations from the desired response, should be limited to the pass band.

2.2 Synthesis Methods Suitable to This Problem

In order to find the one or two synthesis techniques best suited to this problem, it is necessary to form some basis on which the various methods can be judged. First of all, the method should synthesize a network having a purely reactive ladder-type topology. Many synthesis techniques can be eliminated by this constraint alone. A second necessity is that the network can be realized without employing perfect transformers.

One synthesis technique that satisfies these constraints, at least for some transfer functions, is the Darlington Point Impedance Synthesis. A partial answer to the question of when this method will not require perfect transformers is provided by the following theorem:

> The Darlington network is realizable as an unbalanced ladder network without transformers if, and only if, $Z^T(s)$ has its zeros located on the $j\omega$ axis.

A proof of the necessity of this statement is obvious if it is observed that the zeros produced by a reactive ladder network are always zeros of a shunt arm or poles of a series arm. Since the Darlington network is purely reactive, its zeros may therefore lie only on the $j\omega$ axis. Sufficiency of the above theorem will be proven in the section dealing with the Cauer and Guillemin network of the same form.

It must be pointed out that this theorem does not provide a complete answer as to when the Darlington method will not require transformers since there are many other network topologies possible besides the ladder network. However, since the topology of interest to this research has the form of a ladder network, the theorem provides a sufficient answer to the transformer question in this case.

The Darlington synthesis method also has the advantage of being economical in the required number of circuit elements. This comes from the use of the minimum-reactive input impedance in the synthesis calculations. This is a unique quantity for any one specified transfer impedance $Z^T(s)$. One disadvantage of the Darlington method is that it requires rather lengthy calculations compared to many other methods. However, since it satisfies all the criteria in Section 2.1, it will be considered a viable technique in the synthesis of mechanical anti-vibration filters.

While not being able to synthesize all realizable network functions, a method developed by Cauer and Guillemin also appears to satisfy the criteria in Section 2.1. In many cases, the networks synthesized by this method are identical to those obtainable by Darlington's procedure, but it has an advantage in that it is a computationally simpler method.

2.3 Darlington Synthesis Method

Darlington's technique for the synthesis of a reactive network terminated in a resistive load with a specified transfer impedance $Z^T(s)$ makes use of his point impedance synthesis method, published in a 1939 paper.[1] Since the impedance synthesis method is rather

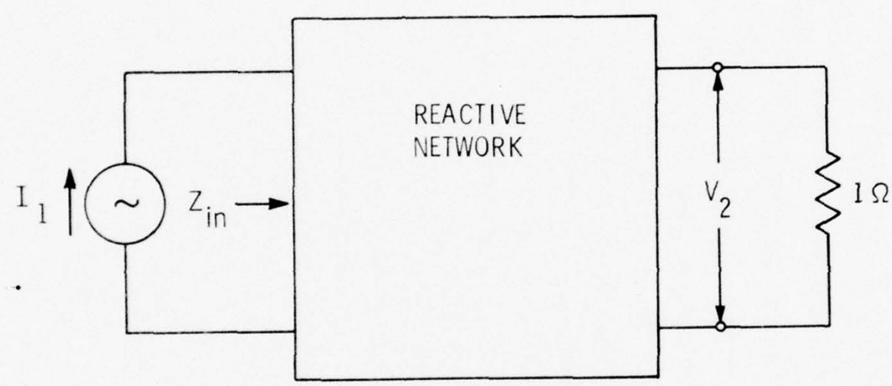


Figure 2.2 Darlington Network

involved and need not be used for the synthesis of low-pass network functions, it will not be discussed here. However, it is useful to look at how the transfer impedance synthesis method works.

As indicated in Figure 2.2, the normalized Darlington network is purely reactive and terminated in a 1Ω load. The squared magnitude of the transfer impedance, $|Z^T(s)|^2$, where $Z^T = V_2/I_1$, is some specified function of frequency and satisfies the appropriate realizability conditions. Letting $R_{in}(\omega) = Re[Z_{in}(s)]$, where $Z_{in}(s)$ is the input impedance of the network, the power P_{in} delivered to the network by the constant current generator is

$$P_{in} = (1/2) \cdot R_{in}(\omega) \cdot |I|^2 .$$

Since the network is purely reactive, the power delivered to the network is equal to the power delivered to the load, i.e., $P_{in} = P_L$, where

$$P_L = (1/2) \cdot |V_2|^2 .$$

Therefore,

$$R_{in}(\omega) = |V_2/I|^2 = |Z^T(s)|^2 \tag{2.1}$$

and it can be seen that the input resistance can be determined from the magnitude of the transfer impedance, $Z^T(s)$.

Darlington's method of synthesizing the network to yield the prescribed $Z^T(s)$ consists of three steps as follows:

1. Determine $R_{in}(\omega)$ as outlined above.

2. Find a realizable $Z_{in}(s)$ such that $Re[Z_{in}(s)] = R_{in}(\omega)$.

3. Use Darlington's impedance synthesis procedure to derive a network having the prescribed $Z_{in}(s)$ terminated in a 1Ω load.

One proof of an input impedance $Z_{in}(s)$ satisfying step 2 is due to Bode.[3] In order for $Z_{in}(s)$ to be a realizable impedance, its poles must be in the left half of the complex frequency or 's' plane. A partial fraction expansion of $|Z^T(s)|^2$ is made and $Z_{in}(s)$ is constructed from the roots corresponding to the left half-plane poles. For example, if $|Z^T(s)|^2$ has poles at $\pm s_1$, $\pm s_2$, and $\pm s_3$, then $Z_{in}(s)$ will have poles at $-s_1$, $-s_2$, and $-s_3$. The $Z_{in}(s)$ so constructed will be minimum-reactive. This method has the disadvantage of, in general, being rather complex, thereby making it unsuitable for numerical purposes.

An alternate method of determining $Z_{in}(s)$ due to Gewertz,[3] using the method of undetermined coefficients, also results in the minimum-reactive input impedance, $[Z_{in}(s)]_{MR}$, a unique quantity. Since $Z_{in}(s)$ must be a realizable function, it can be expressed as the ratio of two polynomials (where the subscripts 'e' and 'o' refer to the even and odd powers of 's', respectively), i.e.,

$$Z_{in}(s) = \frac{P_e(s) + P_o(s)}{Q_e(s) + Q_o(s)} \quad . \tag{2.2}$$

Using Equation (2.2) in connection with Equation (2.1) yields for $s = j\omega$,

$$R_{in}(\omega) = |Z^T(s)|^2 = \frac{P_e(s)\,Q_e(s) - P_o(s)\,Q_o(s)}{Q_e(s)^2 - Q_o(s)^2} . \qquad (2.3)$$

By setting

$$Q_e(s) = a_0 + a_2 s^2 + \ldots ,$$

$$Q_o(s) = a_1 s + a_3 s^3 + \ldots ,$$

$$P_e(s) = b_0 + b_2 s^2 + \ldots$$

and

$$P_o(s) = b_1 s + b_3 s^3 + \ldots$$

in Equation (2.3) and matching the coefficients of $|Z^T(s)|^2$, the a_k and b_k can be determined. The minimum-reactive input impedance is then

$$[Z_{in}(s)]_{MR} = \frac{b_0 + b_1 s + b_2 s^2 + \ldots}{a_0 + a_1 s + a_2 s^2 + \ldots} , \qquad (2.4)$$

where the a_k and b_k are always positive.

A more general input impedance

$$Z_{in}(s) = [Z_{in}(s)]_{MR} + \frac{A_k s}{s^2 + \omega_k^2} + A_\infty s \quad ; \quad A_k > 0 \quad , \quad k = 1,2,3, \ldots$$

could be used since it has the same input resistance, but because it is not minimum-reactive, it would lead to a more complicated circuit. The desired network can now be synthesized from Equation (2.4) in the form of a reactive network terminated in a 1Ω load.

This research is interested only in the design of low-pass ladder filters having a minimum reactive input impedance. These can be constructed using only the Cauer I synthesis techniques[4] which is a simplified special case of the Darlington Impedance Synthesis technique and, therefore, it will not be necessary to go into the detail of the Darlington Impedance synthesis. This input impedance (which has no $j\omega$ axis poles) can be expressed as the continued fraction

$$[Z_{in}(s)]_{MR} = Z_1(s) + \cfrac{1}{\cfrac{1}{Z_2(s)} + \cfrac{1}{Z_3(s) + \cfrac{1}{\cfrac{1}{Z_4(s)} + \cfrac{1}{Z_5(s) + \cdots}}}} \quad (2.5)$$

for the ladder network in Figure 2.3. The coefficients of the continued fraction expansion can be obtained by synthetic division of $[Z_{in}(s)]_{MR}$.

2.4 Cauer and Guillemin Synthesis Method

As stated in Section 2.2, the Cauer[5] and Guillemin[6] synthesis method will usually yield the same network as the Darlington method in the synthesis of low-pass filters. The Cauer and Guillemin method is, however, much simpler computationally. The synthesis method is based on the following two facts:

1. The poles of $Z_{12}(s)$ in a purely reactive network are simple. Moreover, $Z_{11}(s)$ and $Z_{22}(s)$ possess these same poles.

2. Every zero of the mutual impedance $Z_{12}(s)$ in a reactive ladder network must be a zero of a shunt arm or a pole of a series arm.

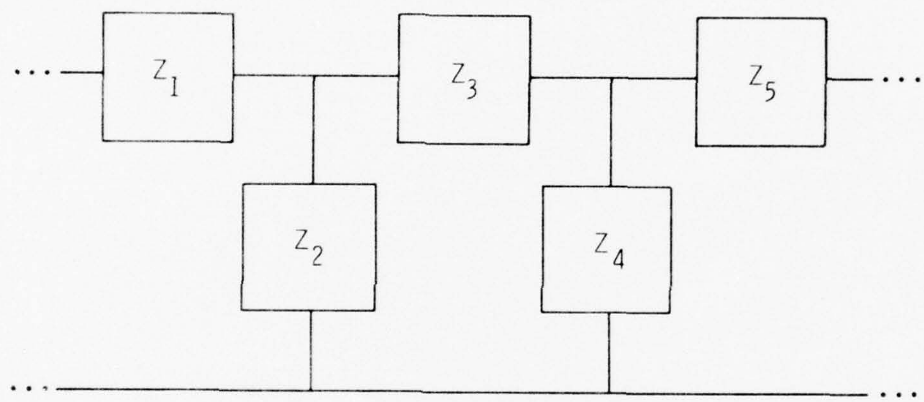


Figure 2.3 Ladder Network

This synthesis method consists of only two steps, as follows:

1. Determine $Z_{12}(s)$ and $Z_{22}(s)$ from the transfer impedance

$$Z^T(s) = \frac{Z_{12}(s)}{1 + Z_{22}(s)} . \tag{2.6}$$

2. Synthesize $Z_{22}(s)$ as a reactive ladder network using the Cauer I method, but in such a way that the zeros of $Z_{12}(s)$ are produced. Cauer suggested that $Z_{12}(s)$ and $Z_{22}(s)$ could be determined by remembering that the ratio of the even to odd parts of a Hurwitz polynomial is a realizable reactance. Since $Z_{22}(s)$ must be a realizable reactance and $Z_{12}(s)$ must be a "reactance-like" function, the appropriate identifications are easily made.

The synthesis of $Z_{22}(s)$ is accomplished by taking one of the zeros of $Z_{12}(s)$ and removing it from $Z_{22}(s)$. Referring to Figure 2.4(a), it is obvious that if $Z_2(s)$ were to have a pole at frequency $s = j\omega_1$ [where ω_1 is the frequency of some zero of $Z_{12}(s)$] that

$$Z_{22}(s)\Big|_{s=j\omega_1} = Z_1(s)\Big|_{s=j\omega_1} .$$

Solving for $Z_{22}(j\omega_1)$ gives the value and type of the reactance $Z_1(s)$. This reactance is then removed from $Z_{22}(s)$ leaving

$$Z_{22}^{(1)}(s) = \frac{1}{1/Z_{22}(s) - 1/Z_1(s)} .$$

Letting $Z_3(j\omega_2)$ be a zero of the shunt arm allows the removal of

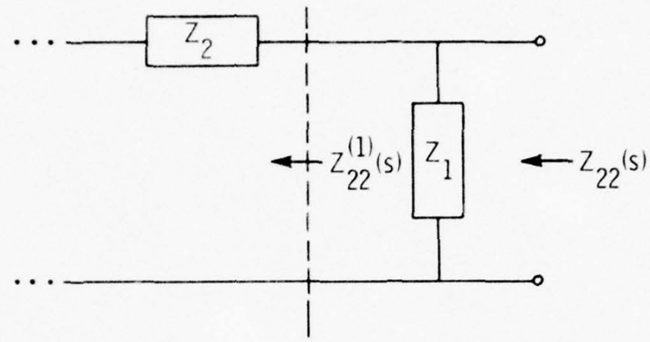
$Z_2(s)$ from $Z_{22}^{(1)}(s)$ leaving [Figure 2.4(b)]

$$Z_{22}^{(2)}(s) = Z_{22}^{(1)}(s) + Z_2(s) \quad .$$

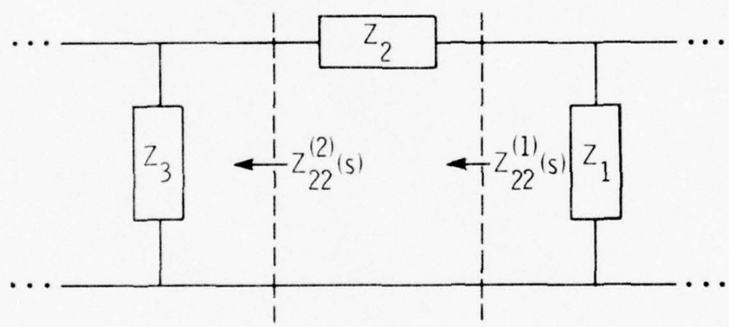
This procedure is carried on until all of the zeros of $Z_{12}(s)$ have been produced in $Z_{22}(s)$. The same method would be used if the leading element, $Z_1(s)$, was a series instead of shunt element.

If there are N different zeros of $Z_{12}(s)$, there will be at most N! different ways of synthesizing the network to give the desired $Z^T(s)$ (within a constant multiplier). This multiplicity of solutions can be advantageous in that it allows the user to select a network meeting some prerequisite such as the least overall inductance (mass) or the reduction of large element values in general. However, even though all of the networks have the same $Z^T(s)$, there is no reason to expect that they will all have the desired topology, in fact, many of them will not.

The fact that the Cauer and Guillemin method can synthesize without transformers any $Z^T(s)$ having its zeros located on the $j\omega$ axis constitutes a proof of the sufficiency of the theorem in Section 2.2 regarding the Darlington synthesis method.



(a)



(b)

Figure 2.4 Partial Synthesis of Reactive Ladder Network

CHAPTER III

METHODS YIELDING A LOW MASS SOURCE

3.1 Modified Transfer Admittance Method (Darlington)

In this section, an example of a network synthesis using the Darlington method will be worked out in detail. The network function to be synthesized is that of a 3-pole Butterworth or maximally flat filter having a transfer admittance

$$|Y^T(s)|^2 = |I_2/V_s|^2 = \frac{H^2}{1 + \omega^6} .$$

This example will be used throughout the next two chapters in order to get a feeling for the problem and to facilitate the comparison of the different methods. The results obtained from this example will then be generalized in Chapter V to provide the basis for the synthesis of other network functions. A theoretical example using the generalized results is presented in Chapter VI.

The network for the 3-pole Butterworth is shown in Figure 3.1 for $H = 1$ yielding a transformerless design and hence, the maximum gain. Note that the mass of the source will be taken as L_1, the lead inductor. As stated in Section 2.1, the final supporting member of the mechanical network must be a spring as shown in Figure 2.1 (a). The electrical equivalent of this spring 'S' and its associated dashpot

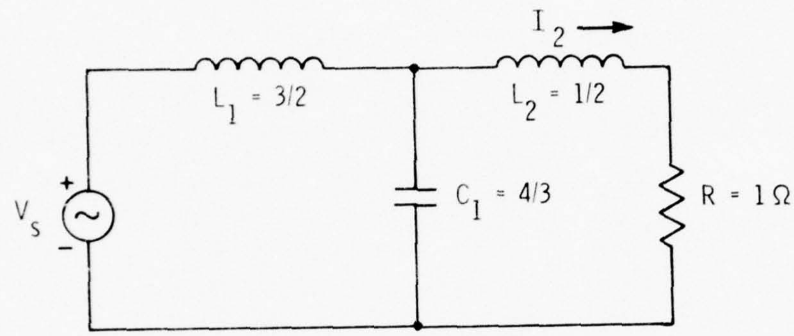


Figure 3.1 3-Pole Butterworth Network

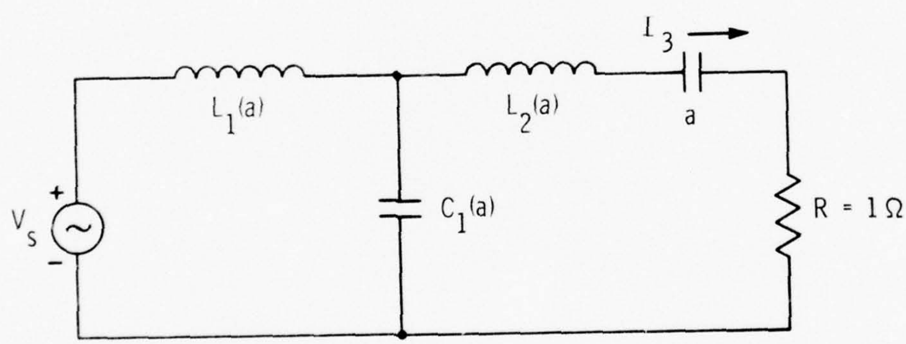


Figure 3.2 Desired Modified 3-Pole Butterworth Network

'R' is a capacitor in series with a resistor ['C' and 'R' in Figure 2.1 (b)]. Since the network in Figure 3.1 does not have a capacitor in series with the load resistor, the transfer admittance will be "modified" (hence, the name of the method) by changing the nature of the load.

It is desired to create the 3-pole Butterworth response across a load consisting of a capacitor in series with a 1Ω resistor. By inserting the series capacitor and specifying that the transfer admittance with respect to the series resistor-capacitor load remain unchanged, the transfer admittance with respect to the load resistor alone has been modified by

$$|I_3/I_2|^2 = \left| \frac{1}{1 + 1/as} \right|^2 .$$

The desired network topology is shown in Figure 3.2 for the modified transfer admittance

$$\begin{aligned} |Y_m^T(s)|^2 &= |Y^T(s)|^2 \cdot |I_3/I_2|^2 \\ &= \frac{H^2}{1+\omega^6} \cdot \left| \frac{1}{1 + 1/as} \right|^2 \\ &= |I_3/V_s|^2 = \frac{-a^2H^2s^2}{1 - a^2s^2 - s^6 - a^2s^8} \ , \end{aligned} \tag{3.1}$$

where the value of the inserted capacitor is 'a'. This network will have the 3-pole Butterworth transfer admittance across the resistor-capacitor load combination instead of across the resistor alone provided that it can be synthesized in this configuration.

The Darlington method will now be used to synthesize this modified transfer admittance. The first step is to determine $Y_{in}(s)$ such that

$$\mathrm{Re}[Y_{in}(s)] = |Y_m^T(s)|^2 .$$

The method of undetermined coefficients results in [using Equation (2.3)] $Q_e^2(s) - Q_o^2(s) = 1 - a^2s^2 - s^6 + a^2s^8$ or

$$a_0^2 = 1 ,$$

$$2a_0a_2 - a_1^2 = -a ,$$

$$2a_0a_4 + a_2^2 - 2a_1a_3 = 0 ,$$

$$2a_2a_4 - 2a_1a_5 - a_3^2 = -1 ,$$

$$a_4^2 - 2a_3a_5 = a$$

and

$$a_5^2 = 0 .$$

With suitable manipulations, the roots are found to be:

$$a_0 = 1 ,$$

$$a_1 = a + 2 ,$$

$$a_2 = 2a + 2 ,$$

$$a_3 = 2a + 1 ,$$

$$a_4 = a$$

and

$$a_5 = 0 .$$

By similar methods, the coefficients of P_e and P_o are found to be

$$b_0 = 0 ,$$
$$b_1 = a^2[1 + (2a + 1)^2/3(a^3 + 2a^2 + 2a + 1)]/a + 2 ,$$
$$b_2 = a^2(2a + 1)^2/3(a^3 + 2a^2 + 2a + 1)$$

and

$$b_3 = a^3(2a + 1)/3(a^3 + 2a^2 + 2a + 1) ,$$

and the minimum reactive input admittance is

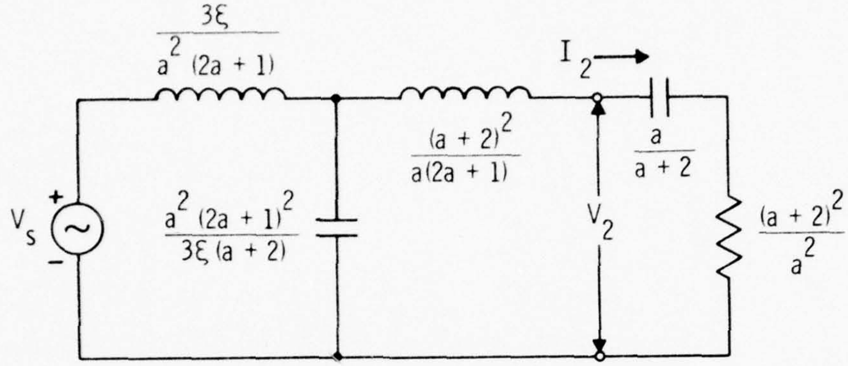
$$[Y_{in}(s)]_{MR} = \frac{\frac{a^2}{a + 2}\left(1 + \frac{(2a + 1)^2}{3\xi}\right)s + \frac{a^2(2a + 1)^2}{3\xi}s^2 + \frac{a^3(2a + 1)}{3\xi}s^3}{1 + (a + 2)s + (2a + 2)s^2 + (2a + 1)s^3 + as^4} \quad (3.2)$$

where $\xi = a^3 + 2a^2 + 2a + 1$.

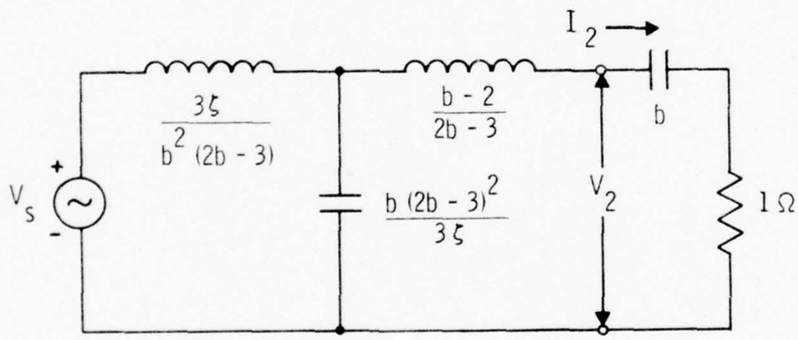
The Cauer I synthesis method can now be used to synthesize Equation (3.2) in the form of a reactive ladder network terminated in a 1Ω resistive load. Doing the synthetic division yields

$$[Y_{in}(s)]_{MR} = \cfrac{1}{\cfrac{3\xi s}{a^2(2a + 1)} + \cfrac{1}{\cfrac{a^2(2a + 1)^2 s}{3\xi(a + 2)} + \cfrac{1}{\cfrac{(a + 2)^2 s}{a(2a + 1)} + \cfrac{(a + 2) + (a + 2)^2 s}{a^2 s}}}}$$

which results in the network shown in Figure 3.3(a). This network satisfies the requirement of having a series resistor-capacitor combination for the load. Figure 3.3(b) shows the network normalized so that the resistor is 1Ω . The value of the series capacitor,



(a) $\xi = a^3 + 2a^2 + 2a + 1$



(b) $\zeta = b^3 - 4b^2 + 6b - 3$

Figure 3.3 Network Synthesizing $|Y_M^T(s)|^2 = \dfrac{-a^2s^2}{1 - a^2s^2 - s^6 + a^2s^8}$

however, is not 'a' but $b = a + 2$. This change in the value of the series capacitor from 'a' to $a + 2$ means that the exact 3-pole Butterworth response cannot be obtained with the circuit containing a series capacitor. However, in the limit as 'a' or 'b' $\to \infty$, all of the elements in Figure 3.3 return, as expected, to the values shown in Figure 3.1. The lower limit of 'b' is predictable in advance as will be shown in Section 5.3.

3.2 Modified Transfer Admittance Method (Cauer and Guillemin)

As stated in Section 2.2, the Cauer and Guillemin method in many cases will yield the same network as the Darlington method but requires fewer calculations. Generating the a_k , b_k , and doing the synthetic division to find the coefficients of the continued fraction expansion of $Y_{in}(s)$ is not an easy task. This section will show how a network having the same transfer admittance, $Y_M^T(s)$, as in Section 3.1 may be generated using the Cauer and Guillemin technique.

Since the squared magnitude of the transfer admittance $|Y_M^T(s)|^2$ is the specified quantity, $Y_M^T(s)$ must be determined first. Having found $Y_M^T(s)$ to be

$$Y_M^T(s) = \frac{Has}{1 + (a + 2)s + (2a + 2)s^2 + (1 + 2a)s^3 + as^4} ,$$

where the denominator contains the left-half plane roots of the denominator of $|Y_M^T(s)|^2$, the admittances $Y_{12}(s)$ and $Y_{22}(s)$ are by inspection [using the admittance form of Equation (2.6)]:

$$Y_{12}(s) = \frac{Has}{1 + (2 + 2a)s^2 + as^4}$$

and

$$Y_{22}(s) = \frac{(2 + a)s + (1 + 2a)s^3}{1 + (2 + 2a)s^2 + as^4} .$$

It can be seen that $Y_{12}(s)$ has a simple zero at $s = 0$ and a triple zero at $s = \infty$. Referring to Figure 3.4(a), we see that by letting $Y_2(s)$ be a zero at $s = \infty$;

$$Y_1(s)\Big|_{s = \infty} = Y_{22}(s)\Big|_{s = \infty} .$$

The residue of $Y_{22}(s)$ at $s = \infty$ is $(1 + 2a)/a$ which corresponds to a series inductor of value $a/(1 + 2a)$. Removing the inductor from $Y_{22}(s)$ yields

$$1/Y_{22}(s) - as/(1 + 2a) = 1/Y_{22}^{(1a)}(s)$$

and

$$Y_{22}^{(1a)}(s) = \frac{(1 + 2a)^2 s^3 + (1 + 2a)(a + 2)s}{(3a^2 + 4a + 2)s + (1 + 2a)} .$$

It can be seen that $Y_{22}^{(1a)}(s)$ still has a zero at $s = 0$ that can be removed. The residue of $Y_{22}^{(1a)}(s)$ at $s = 0$ is $a + 2$ and, therefore, a series capacitor can be removed from $Y_{22}^{(1a)}(s)$ leaving

$$Y_{22}^{(1)}(s) = \frac{(a + 2)(1 + 2a)^2 s^4 + (1 + 2a)(a + 2)s^2}{(3a^3 + 10a^2 + 8a + 3)s^3} .$$

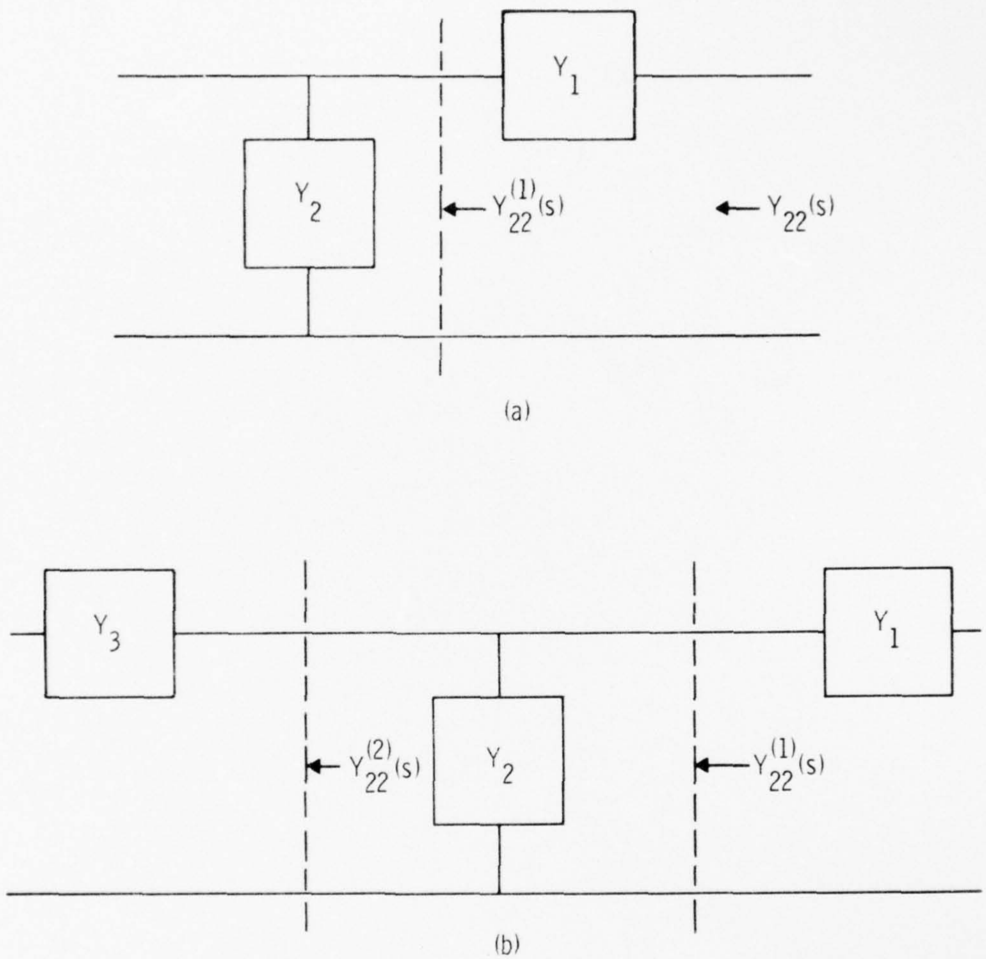


Figure 3.4 Cauer and Guillemin Synthesis of $Y_M^T(s)$

A pole at $s = 0$ (shunt capacitor) can be removed having a value of

$$\frac{(a + 2)(1 + 2a)^2}{3a^3 + 10a^2 + 8a + 3} ,$$

leaving

$$Y_{22}^{(2)}(s) = \frac{(1 + 2a)(a + 2)^2}{(3a^3 + 10a^2 + 8a + 3)s} ,$$

a series inductor. By letting $a + 2 = b$, the same network as shown in Figure 3.3(b) has been synthesized. Since there are two zeros of $Y_{12}(s)$, there is one other way of synthesizing $Y_M^T(s)$. The other synthesis will place a capacitor in series with the generator instead of the load resistor and therefore is not usable.

3.3 Response Characteristics

In the evaluation of the performance of an anti-vibration mechanical filter driven by a constant force generator, transmissability or the ratio of the driving force to the force exerted on the foundation is the quantity of most interest. The following section will analyze the response characteristics of the 3-pole Butterworth network synthesized in Sections 3.1 and 3.2.

The network synthesized using the modified transfer admittance $|Y_M^T(s)|^2$ and the corresponding mechanical network are shown in Figures 3.5(a) and (b), respectively. The synthesis method requires the mass of the vibrating object to be equal to M_1. If the object mass is less than M_1, additional mass can be added until the total mass is equal to M_1, or, the values of the other elements may be altered by

a constant multiplier α , such that

$$\alpha \cdot M_1 = M_{object} .$$

Changing the impedances of all the elements in a network by a constant multiplier will not alter the overall response of the network.

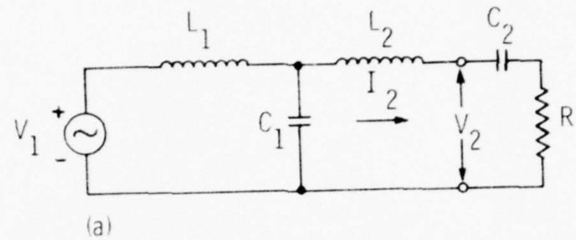
The object mass M_1 is seen to be an integral part of the T-filter consisting of elements M_1 , C_1 , and M_2 . The response $|V_2/V_1|^2$ of this T-filter is computed as follows; the transfer admittance of the network in Figure 3.5(a) is

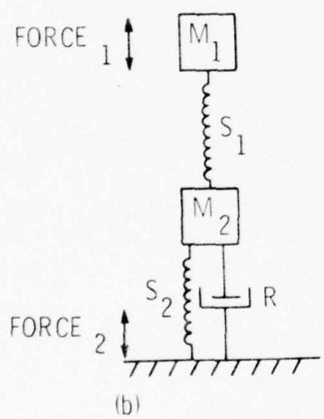
$$|Y_M^T(s)|^2 = \frac{-(b-2)^2 H^2 s^2}{1-(b-2)^2 s^2 - s^6 + (b-2)^2 s^8} \tag{3.3}$$

for normalization to $b = (a + 2)$ and $R = 1\Omega$. The voltage V_2 across the load is equal to the impedance of the load multiplied by the current flowing through it; therefore, $V_2 = I_2 Z$, where $Z = 1 + 1/bs$. The squared magnitude of the response function V_2/V_1 can then be shown to be:

$$|V_2/V_1|^2 = |Y_M^T(s)|^2 \cdot |Z|^2 = \frac{1 + b^2\omega^2}{1 + (b-2)^2\omega^2 + \omega^6 + (b-2)^2\omega^8} , \tag{3.4}$$

where $H = (a + 2)/a$ for a transformerless design. This has been plotted in Figure 3.6 on a decibel scale for values of $b = 4$ through 12 in increments of 2 over the range $0 \le \omega \le 2$. For the unmodified network, $|Z|^2 = 1$. Therefore, $|V_2/V_1|^2 = |Y^T(s)|^2 \cdot |Z|^2 = 1/(1 + \omega^6)$ where $H = 1$.





$$L_1 = M_1 = \frac{3(b^3 - 4b^2 + 6b - 3)}{b^2\,(2b - 3)}$$

$$\frac{1}{S_1} = C_1 = \frac{(2b - 3)^2\, b}{3(b^3 - 4b^2 + 6b - 3)}$$

$$L_2 = M_2 = \frac{b - 2}{2b - 3}$$

$$\frac{1}{S_2} = C_2 = b$$

$$R = 1$$

Figure 3.5 Analagous 3-Pole Butterworth Networks (Darlington)

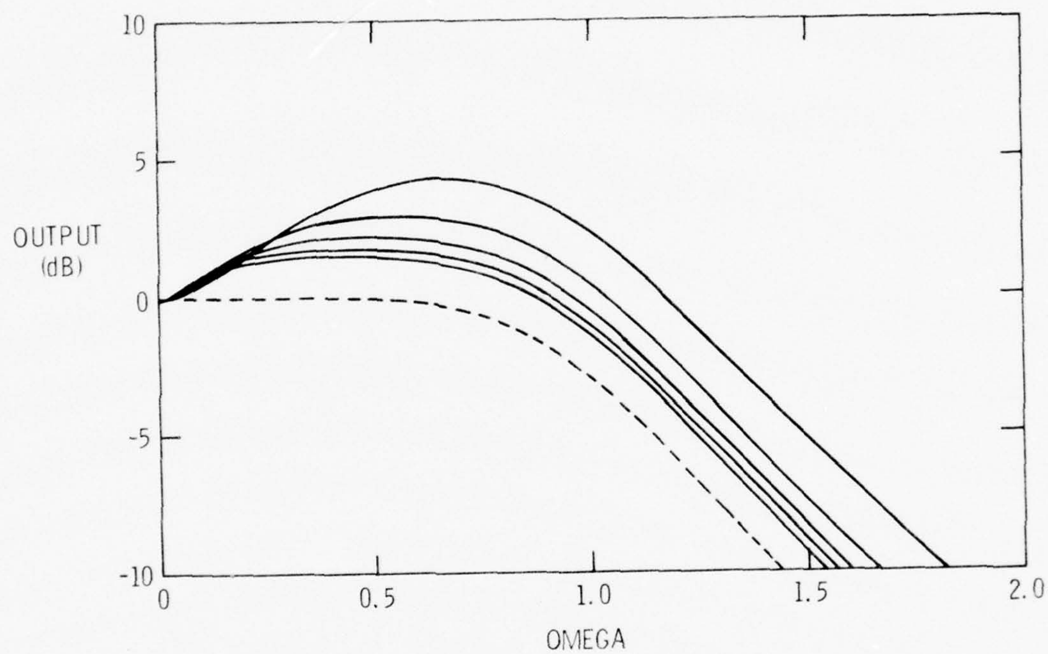


Figure 3.6 $|V_2/V_1|^2$ vs. Omega for the Modified Transfer Admittance Method

The most obvious thing to notice is that the response function of the modified network does not behave in the same way as the unmodified network (shown by the dashed line). The response of the unmodified network drops at 18 dB per octave for $\omega > 1$. The rise in response varies from approximately 5 dB at $\omega = 1$ for $b = 4$ to 1.5 dB for $b = 12$. For values of ω greater than 1, the difference in response is fairly constant.

Since all the curves describing the modified network are higher in amplitude than that of the unmodified network, the 3 dB down point has moved up in frequency. For $C_2 = 4$ $(a = 2)$, the 3 dB down point has moved to approximately $\omega \approx 1.35$, almost one third of an octave increase. This effect lessens as the value of C_2 becomes larger.

3.4 Network Limitations

As with any kind of synthesis technique, the methods used here have certain limitations and restrictions. Some of these restrictions have already been mentioned in Sections 2.1 and 2.2. However, there are also limitations and restrictions in the networks synthesized by these techniques. These shortcomings and their effects on the applications and usefulness of these filters will now be discussed.

Referring back to Section 3.1, notice that when the synthesized 3-pole Butterworth network [Figure 3.3(a)] had been normalized to a load resistance of 1Ω the value of the load capacitor became $b = a + 2$, where 'a' was the value of the capacitor used to modify the transfer admittance. Therefore, if the value of the modifying capacitor had been $a = 0$, the resulting load capacitor would have a value of $b = 2$.

Since 'a' cannot be zero (this results in zero or infinite element values), there is a lower limit on the size of the load capacitor, i.e., $b > 2$ (for $R = 1\Omega$).

In terms of the mechanical system [Figure 3.5(b)], the supporting spring S_2 cannot have a greater stiffness constant than 0.5 N/m in the network normalized such that the dashpot has a value of 1 Mech. Ω and the cutoff frequency is $\omega = 1$. With such a small stiffness constant, the rest of the system must be very light so that a long spring is not required to support the static load.

As an example, for $b = 4$ and $R = 1\Omega$, the 3-pole Butterworth mechanical network [Figure 3.5(b)] has the element values

$$
\begin{aligned}
M_1 &= 0.78 \text{ kg} \\
S_1 &= 3.15 \text{ N/m} \\
M_2 &= 0.4 \text{ kg} \\
S_2 &= 0.25 \text{ N/m} \\
R &= 1 \text{ kg/s} = 1 \text{ Mech. } \Omega .
\end{aligned}
$$

The total force due to M_1 and M_2 alone on the supporting spring S_2 is over 11 Newtons which means that the spring must be over 46 meters in length just to support the static load. Obviously, this is not a satisfactory situation. Since the element values of the network are all (except the dashpot) functions of the supporting spring S_2, a renormalization of the element values to some different value of the dashpot will make no difference. As the value of the dashpot goes up, so will the value of the masses and the stiffness of the springs.

Remember, however, that this network has been normalized to a cutoff frequency of $\omega = 1$ or $f = 0.159$ Hz, an extremely low value. By raising the cutoff frequency to $\omega = 1000$ or $f = 159$ Hz, for example, the value of the masses M_1 and M_2 will be reduced by a factor of 1000 and the spring stiffnesses will be increased by a factor of 1000. Using these new values, the supporting spring S_2 is compressed a total of 0.046 millimeters by the static load of masses M_1 and M_2 , quite a large change.

Notice that this method favors low source masses. For a 3-pole Butterworth network normalized to a cutoff at $\omega = 1$ and a load of $R = 1\Omega$, the source mass could not be larger than 1.5 kilograms or lower than 0.75 kilograms, depending on the value chosen for the stiffness of the support spring S_2. As the source mass becomes larger, the springs must be increased in stiffness proportionately. Eventually, the spring stiffness would become too large to work with; at this point, the "Capacitor Shift" method, to be described in the next chapter, has the advantage in that it works better with larger source masses.

While the subject of lossy elements, particularly lossy or damped springs, will not be exhaustively treated in this paper, it is possible to give a qualitative description of their effect on the network performance. Referring to Figure 3.5(b), it can be seen that there is only one spring (S_1) that has no loss associated with it. Since any real spring will be somewhat lossy, the response curve as shown in Figure 3.6 will be affected to some extent.

Any loss inherent in spring S_1 will show up as a resistance in series with capacitor C_1 in Figure 3.6(a). This resistance will

be negligible at low frequencies where the impedance of capacitor C_1 is large. At high frequencies, however, the resistance can no longer be considered negligible; in fact, its impedance will eventually become larger than that of the capacitance. When this happens, the response curve will cease to drop at 18 dB per octave since the capacitor cannot create a zero in the shunt arm. Instead, the response will drop at 12 dB per octave for these higher frequencies.

The change in the response characteristics will, of course, not be abrupt and over what frequencies this change takes place will be determined by the extent of the damping. As the damping becomes larger, the deviation from the desired response will occur at ever lower frequencies.

CHAPTER IV

METHODS YIELDING HIGH MASS SOURCES

4.1 Capacitor Shift Method

The two synthesis methods discussed so far result in a low source mass network driven by an ideal voltage generator. There may, however, be occasions when the source is a large mass moving with a constant velocity. The following method is applicable to the design of a network driven by this type of source. We will start by assuming the source mass to be infinite and then consider how the response is modified when the mass is made large but finite.

The 3-pole Butterworth network can be designed as a π-network driven by an ideal constant current source. If an additional finite impedance is inserted in series with the source, the current into the original π-filter will remain the same. In other words, the source will continue to generate the same current no matter what the load.

With this in mind, a series capacitor can be inserted between the generator and the first shunt capacitance of the 3-pole Butterworth network in Figure 4.1(a) and not affect the response of the circuit [Figure 4.1(b)]. This series capacitor can now be "shifted" through the network until it is in series with the resistor, its desired position.

The following impedance transformation is the method by which this is accomplished. The network shown in Figure 4.2(a) is specified to have the same input impedance as the network in Figure 4.2(b) and

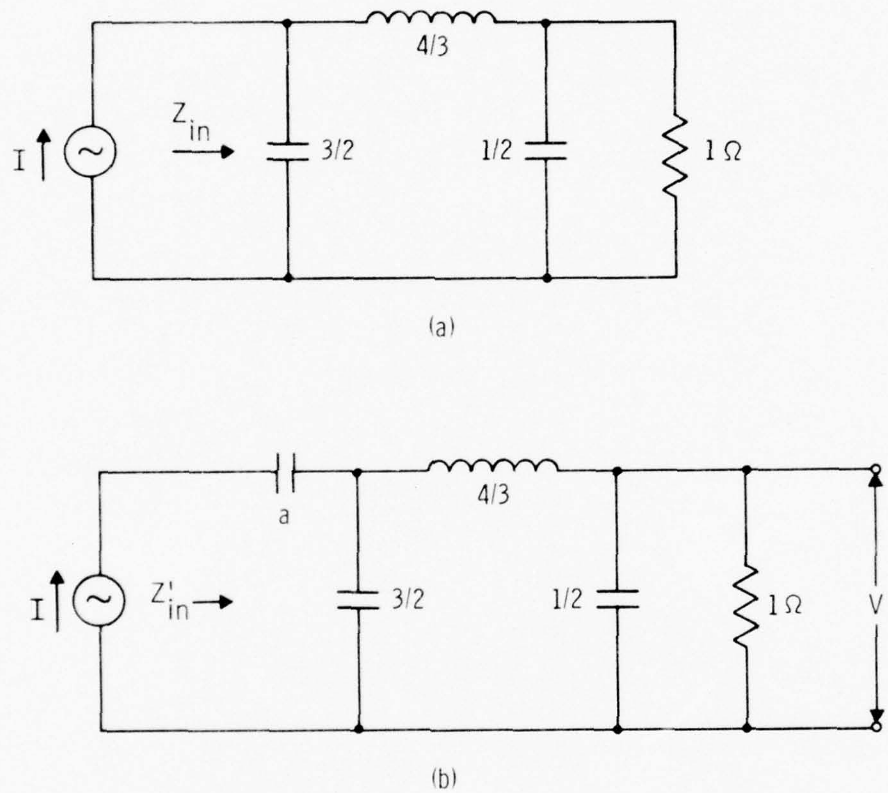


Figure 4.1 Equivalent 3-Pole Butterworth Networks

three equations relating the elements in each network are found. They are:

$$C_1 = 1/(1/C_3 + 1/C_4) \quad , \tag{4.1a}$$

$$C_2 = C_3 - C_1 \tag{4.1b}$$

and

$$Z_A = (C_3 + C_4)^2 \, Z_B/C_3^2 \quad . \tag{4.1c}$$

It can be seen that Figure 4.1(b) corresponds to the circuit shown in Figure 4.2(b), where $C_3 = a$, $C_4 = 3/2$, and Z_B is the rest of the network from the series inductor to the load resistor. The first time through the transform from (b) to (a) results in the circuit in Figure 4.3(a). The series capacitor has now been "shifted" past the first shunt capacitor and all of the original element values have now become functions of the inserted capacitor. The positions of the series inductor and capacitor can now be interchanged and the process repeated to shift the capacitor in series with the load resistor, its desired position. This second transformation results in the network shown in Figure 4.3(b). Normalizing the series capacitor to a value of $b = a + 2$ and the resistance to 1Ω results in the network shown in Figure 4.3(c). Care must be taken, however, in the normalization of these networks.

When transforming from the original network [Figure 4.1(b)] to the network in Figure 4.3(b), the power dissipated in the resistance will remain the same. Since the resistance in Figure 4.3(b) is $(a + 2)^2/a^2$, the output voltage must therefore be $(a + 2)/a$ times the voltage V in Figure 4.1(b). Normalization of the load resistor

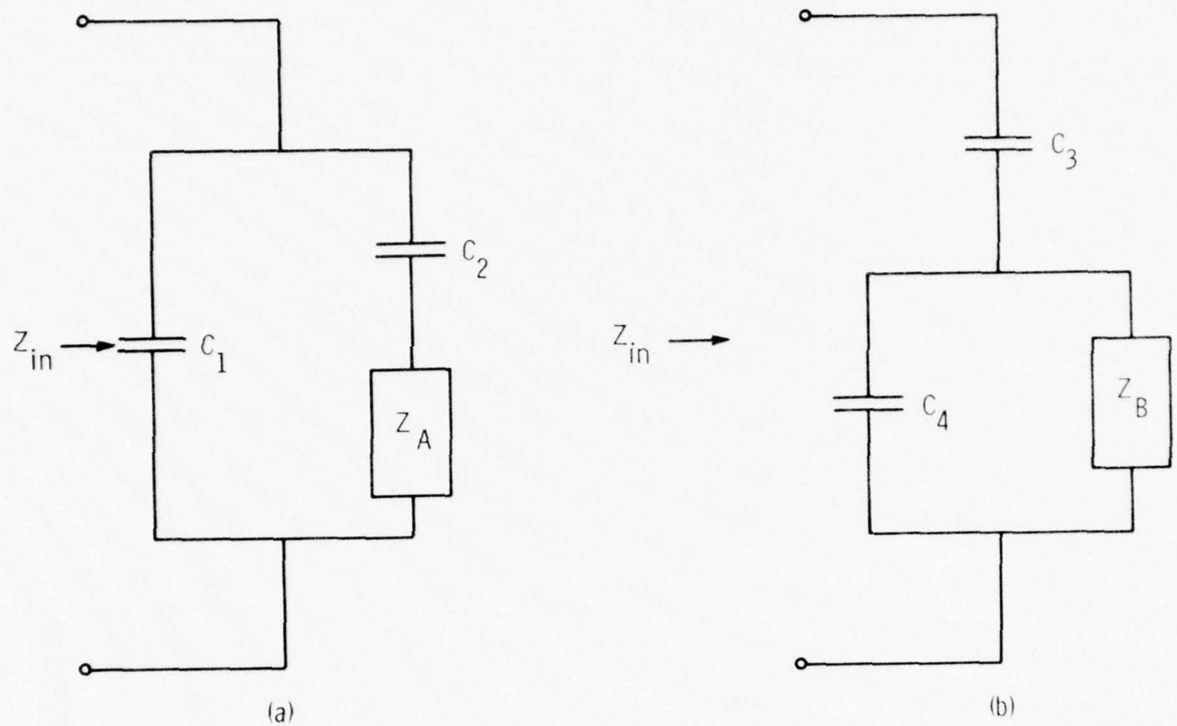


Figure 4.2 Impedance Transformation Circuits

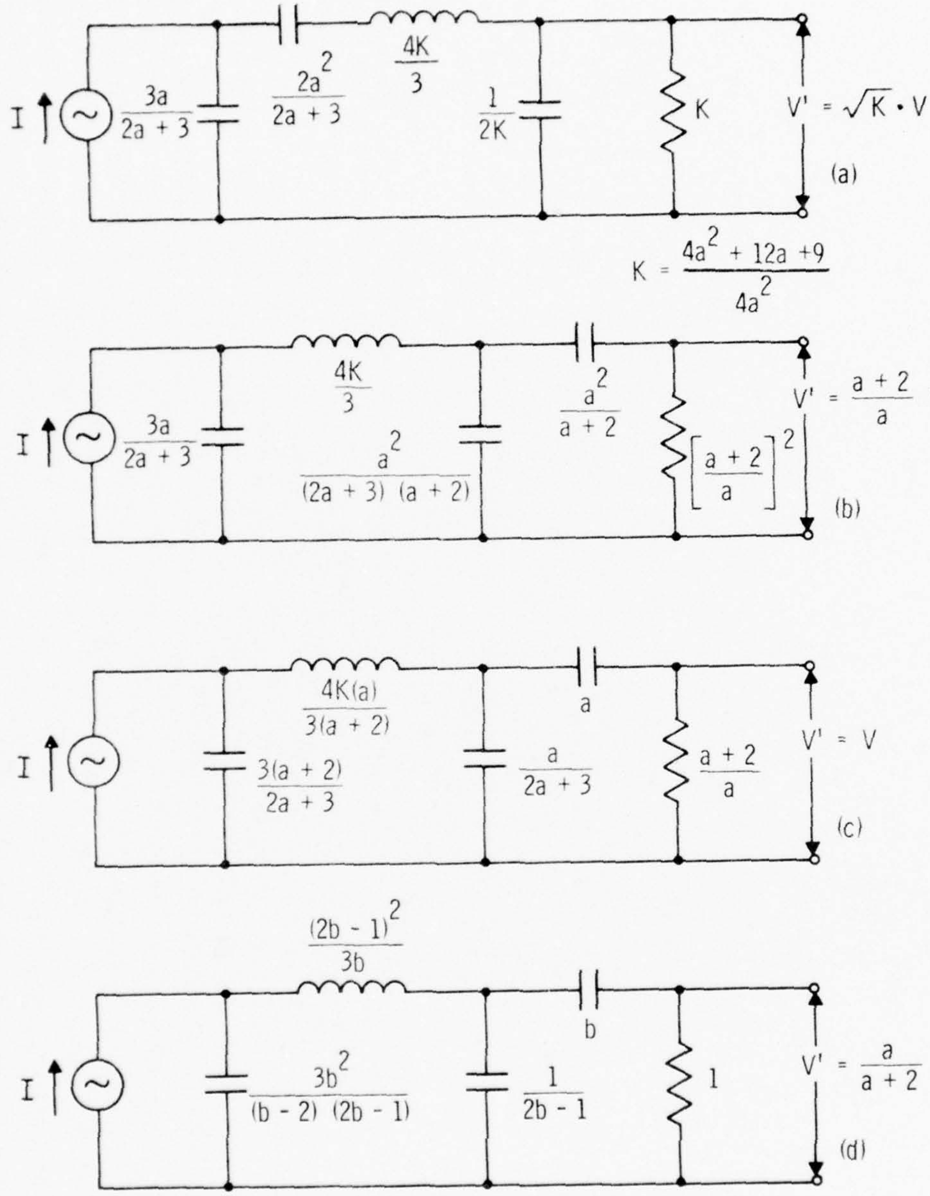


Figure 4.3 Capacitor Shift Circuit Transformations

to 1Ω [as in Figure 4.3(c)] causes the output voltage to be reduced by the factor $(a + 2)^2/a^2$ or $V' = V\, a/(a + 2)$. The magnitude of the output voltage is important in determining the network response as will be demonstrated in Section 4.3.

This method is computationally quite simple and results in a network having a current generator or high impedance for the source. As expected, the element values return to those of a 3-pole Butterworth as $b \to \infty$.

4.2 Capacitor Shift Method (Cauer and Guillemin)

The Cauer and Guillemin synthesis method can also be used to generate the same network as derived by the Capacitor Shift method in Section 4.1. In this case, the synthesis method will use an unmodified transfer impedance function rather than the modified transfer admittance function used in Section 3.2. Having found the transfer impedance function for the 3-pole Butterworth to be

$$Z^T(s) = \frac{H}{1 + 2s + 2s^2 + s^3} ,$$

the impedances Z_{12} and Z_{22} are [using Equation (2.6)]:

$$Z_{12}(s) = \frac{H}{2s + s^3}$$

and

$$Z_{22}(s) = \frac{1 + 2s^2}{2s + s^3} .$$

In order to achieve a series resistor-capacitor load, the first step is

to remove a series capacitor of value 'b', the limits of which become apparent later. This results in

$$Z_{22}^{1}(s) = Z_{22}(s) - 1/bs = \frac{b(1 + 2s^2) + (2 + s^2)}{bs(2 + s^2)}$$

or

$$Z_{22}^{1}(s) = \frac{1 + (2b - 1/b - 2)s^2}{2b/(b - 2) + bs^3/(b - 2)} \quad . \qquad (4.2)$$

The triple zero at $s = \infty$ has yet to be synthesized. Since the rest of the network is in the form of a purely reactive ladder network, the Cauer I synthesis method can be used to finish the synthesis of this network. Letting $Z_{22}^{1}(s)$ be the continued fraction expansion,

$$Z_{22}^{1}(s) = \cfrac{1}{C_2 s + \cfrac{1}{M_1 s + \cfrac{1}{C_1 s}}} \ ,$$

then

$$Z_{22}^{1}(s) = \frac{1 + M_1 C_1 s^2}{(C_1 + C_2)s + M_1 C_1 C_2 s^3} \quad . \qquad (4.3)$$

By equating Equations (4.2) and (4.3), the values of C_1 , M_1 , and C_2 are determined to be

$$C_1 = \frac{3b^2}{(b - 2)(2b - 1)} \ ,$$

$$M_1 = \frac{(2b - 1)}{3b^2}$$

and

$$C_2 = \frac{b}{2b - 1} \ ,$$

where $C_3 = b$ and $R = 1\Omega$. These element values are the same as generated by the Capacitor Shift method for the network shown in Figure 4.3(b).

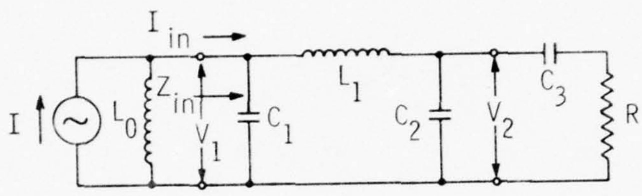
Recall, though, that the size of 'b' has been left arbitrary to this point. For the elements to be real, they must be positive and this constrains 'b' to be within certain limits. These limits are, in this case, $2 \leq$ 'b' $\leq \infty$. It is possible, however, to predict the lower limit of 'b' as will be shown in Section 5.5.

4.3 Response Characteristics

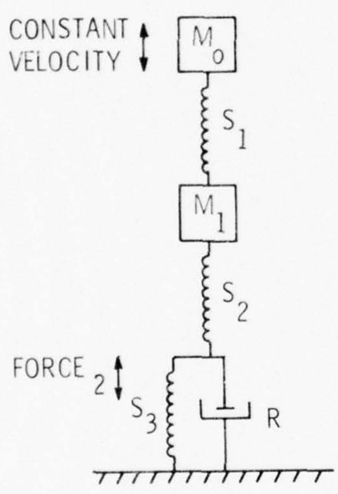
Recall that the networks shown in Figure 4.3 are driven by infinite source masses. This is due to the fact that constant current sources are analagous to infinite masses moving with a constant velocity. Since any real source is finite, an inductor must be placed across the current source. The magnitude of this inductor is equal to the magnitude of the source mass, M_0 . The 3-pole Butterworth network synthesized using the Capacitor Shift method [Figure 4.3(c)] and its corresponding mechanical network are shown in Figures 4.2(a) and (b), respectively, with the modification of a finite source mass.

Although the quantity of interest is $|V_2/V_1|^2$, the force transmissability of the network, it is worthwhile to first calculate $|V_2/I|^2$, the transfer impedance of the network. While $|V_2/I|^2$ may be readily calculated and plotted, a better understanding comes from making a simplistic analysis of the network.

The network shown in Figure 4.1(b) has a magnitude squared transfer impedance



(a)



(b)

$$\frac{1}{S_1} = C_1 = \frac{3(a+2)}{2a+3}$$

$$L_1 = M_1 = \frac{4a^2 + 12a + 9}{3a(a+2)}$$

$$\frac{1}{S_2} = C_2 = \frac{2}{2a+3}$$

$$\frac{1}{S_3} = C_3 = a$$

$$R = \frac{a+2}{a}$$

Figure 4.4 Analagous 3-Pole Butterworth Networks (Capacitor-Shift)

$$|Z^T(s)|^2 = |V'/I|^2 = 1/(1 + \omega^6)$$

and an input impedance

$$Z'_{in}(s) = \frac{1 + (a + 2)s + (2 + 4a/3)s^2 + (1 + 2a/3)s^3}{as(1 + 2s + 2s^2 + s^3)} .$$

The transformed network [Figure 4.3(b)] must be virtue of the impedance transformations Figures 4.1(a) through (c), have the same input impedance. It does not, however, have the same transfer impedance. It has already been pointed out that, for Figure 4.3(b),

$$V' = \frac{a + 2}{a} \cdot V$$

and therefore,

$$|Z^T(s)|^2 = |V'/I|^2 = \frac{(a + 2)^2/a^2}{1 + \omega^6} .$$

By normalizing to a resistance of $(a + 2)/a$ (the square root of the transformed resistance), the magnitude squared transfer impedance is once again $|Z^T(s)|^2 = 1/(1+ \omega^6)$, but now the input impedance is

$$Z'_{in}(s) = \frac{a}{a + 2} \cdot Z_{in}(s)$$

as shown in Figure 4.3(c).

The addition of the inductor M_0 shown in Figure 4.4(a) causes a low frequency resonance in the transfer impedance function. This

resonance is between the inductor M_0 and the capacitor C_3 and is caused by the fact that the π-filter consisting of elements C_1, M_1, and C_2 no longer sees a constant current source. Letting

$$V_1 = I_{in} \cdot Z'_{in}(s) = (I - I_{in}) \cdot M_0 s \quad ,$$

then

$$I_{in}[M_0 s + Z'_{in}(s)] = I \cdot M_0 s$$

or

$$\frac{I_{in}}{I} = \frac{M_0 s}{M_0 s + Z'_{in}(s)} \quad .$$

Since the transfer impedance of the original network is

$$|Z^T(s)|^2 = |V'/I_{in}|^2 = 1/(1 + \omega^6) \quad ,$$

the total magnitude squared transfer impedance with the additional inductor M_0 is

$$|Z^T(s)|^2 = |V'/I|^2 = \frac{1}{1 + \omega^6} \cdot \left| \frac{M_0 s}{M_0 s + Z'_{in}(s)} \right|^2 \quad . \tag{4.4}$$

However, the quantity of interest is $|V_2/I|^2$, and thus Equation (4.4) is modified by the factor

$$\left| \frac{V_2}{V'} \right|^2 = 1 + \frac{1}{(a + 2)^2 \omega^2}$$

to yield

$$\left| \frac{V_2}{I} \right|^2 = \frac{1}{1 + \omega^6} \cdot [1 + 1/(a + 2)^2 \omega^2] \frac{M_0{}^2 \omega^2}{|M_0 s + Z'_{in}(s)|^2} \quad . \tag{4.5}$$

Equation (4.5) is plotted in Figures 4.5(a) and (b) for $M_0 = 10$ and 100, respectively, for $C_3 = 2$ through 10 in increments of 2. The curves all converge to the desired response (dotted line) by the cutoff frequency $(\omega = 1)$. As expected, the resonant peak moves down in frequency as M_0 increases. There is also a slight dependence on the final capacitance C_3, the peak moving lower in frequency as the capacitance increases.

In order to provide a basis for comparison between the Capacitor Shift and Modified Transfer Admittance methods it is necessary that the networks have the same source configuration. Assume that the source mass to be isolated is known to be moving with a constant velocity but is too large for use in the modified Transfer Admittance method. The Capacitor Shift method can be used in this case to synthesize a network having the proper source configuration. The 3-pole Butterworth network synthesized by the Capacitor Shift method [Figure 4.4(a)] is altered to obtain the source configuration of a mass driven by a force generator.

In order to do this, the parallel combination of the current source and the inductor M_0 must be replaced with a series combination of a voltage source and the same inductor M_0. Figure 4.6 shows the two equivalent sources which are related by

$$V_s = I_{in} \cdot [M_0 s + Z'_{in}(s)] \ .$$

Therefore, replacing the current generator by the voltage generator (Figure 4.7), the magnitude squared voltage transfer function is

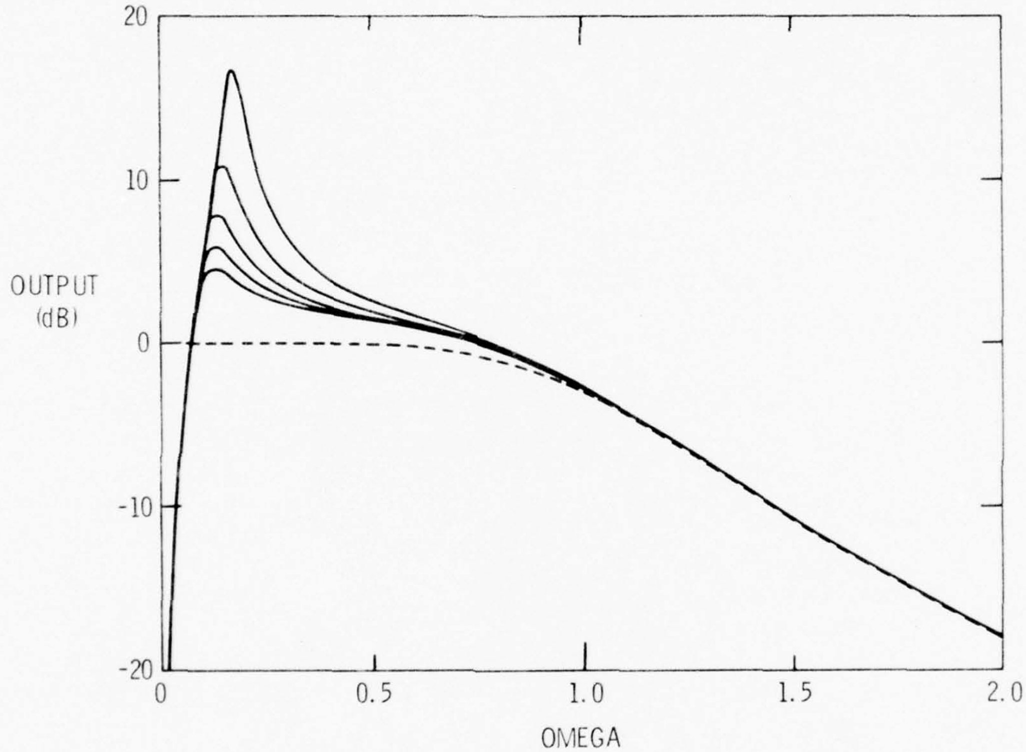


Figure 4.5(a) Response $|V_2/I|^2$ of Network in Figure 4.4(a)

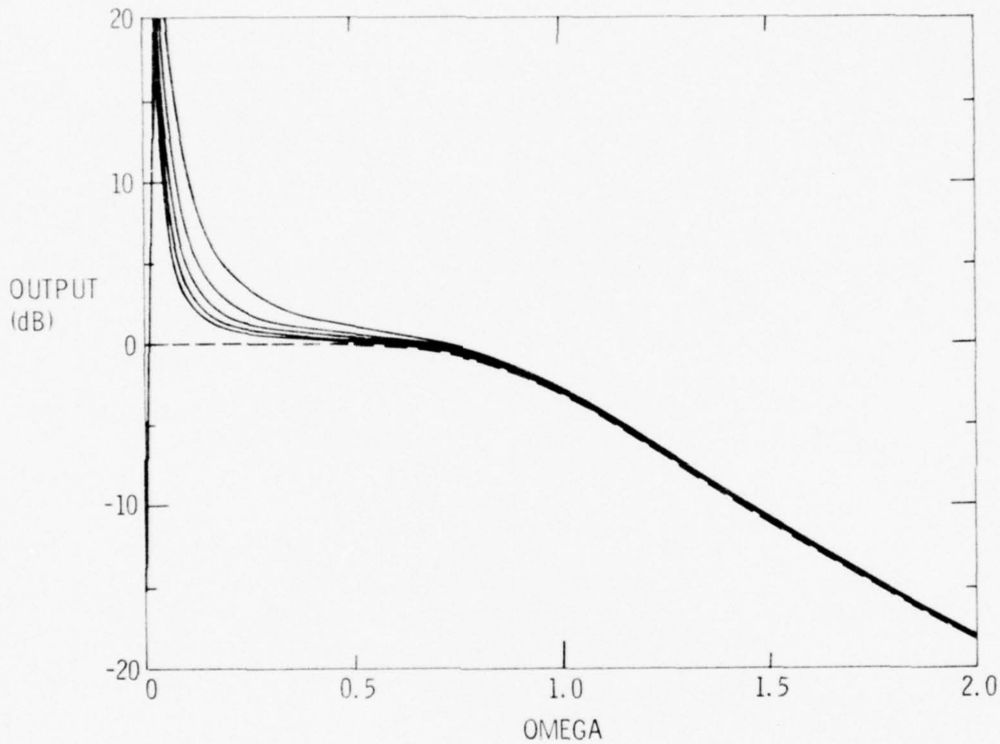
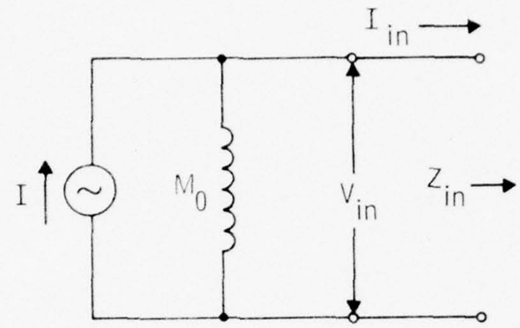
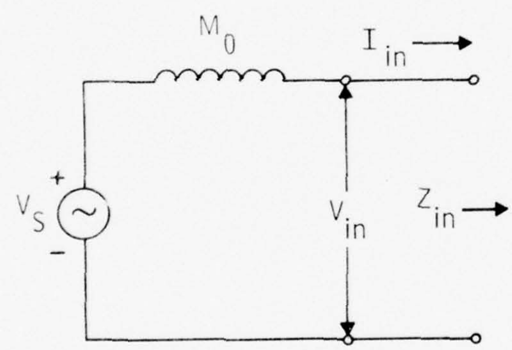


Figure 4.5(b) Response $|V_2/I|^2$ of Network in Figure 4.4(a)



(a)



(b)

$$V_S = I_{in}(M_0 S + Z_{in}(s))$$

Figure 4.6 Equivalent Source Configurations

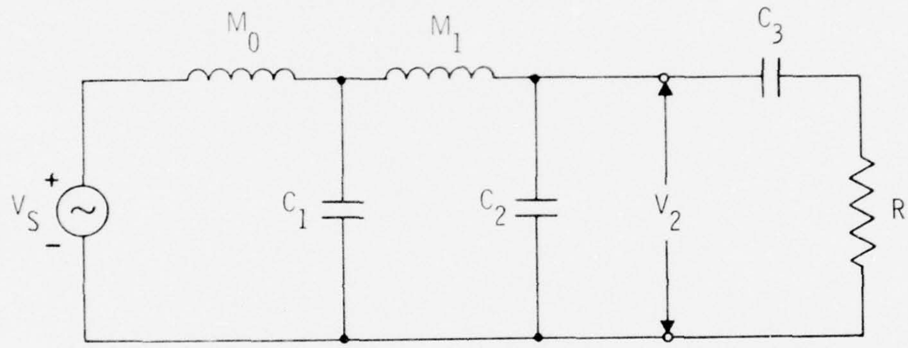


Figure 4.7 Force Drive Network

$$\left|\frac{V_2}{V_s}\right|^2 = \frac{1}{1+\omega^6} \cdot [1 + 1/(a+2)^2\omega^2] \cdot \frac{1}{|M_0 s + Z'_{in}(s)|^2} . \quad (4.6)$$

Equation (4.6) is plotted in Figures 4.8(a) and (b) for $M_0 = 10$ and 100 respectively for values of $C_3 = 2$ through 10 in increments of 2.

Notice that the force transmissibility drops 6 dB per octave faster than the desired 3-pole Butterworth response. Also, the actual response curve is separated from the desired response by quite a large amount. These differences are caused by the last term in Equation (4.6). The extra 6 dB per octave rolloff can be accounted for in that the series source mass adds an extra pole to the response function. What originally was a 3-pole response is now a 4-pole response. The constant offset at higher frequencies is equal to $10 \cdot \text{Log}(1/M_0^2)$ and is fairly constant above the cutoff frequency since the value of $Z'_{in}(s)$ is approximately 1 at $s = 1$.

Since the difference between the desired and resultant responses is known, the response that was originally used [that is, the response of the network in Figure 4.1(a)] can be altered so that the final network will have the desired response. For example, the Capacitor Shift method would require an original response function of a 2-pole Butterworth in order to produce approximately 3-pole Butterworth response after the source change. Or, the cutoff frequency of the original response can be raised so that the resultant curve will meet the desired criteria at the correct frequency. This is shown diagramatically in Figure 4.9 where the resultant response has a higher cutoff frequency than the original response.

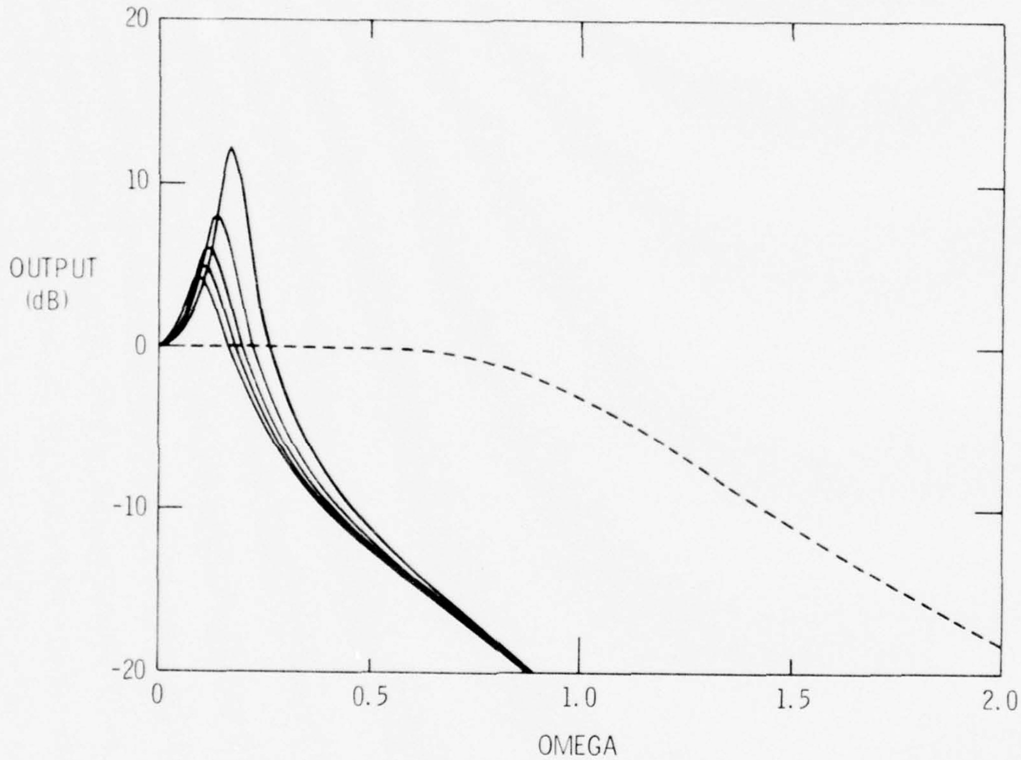


Figure 4.8(a) $|V_2/V_s|^2$ for Network in Figure 4.7

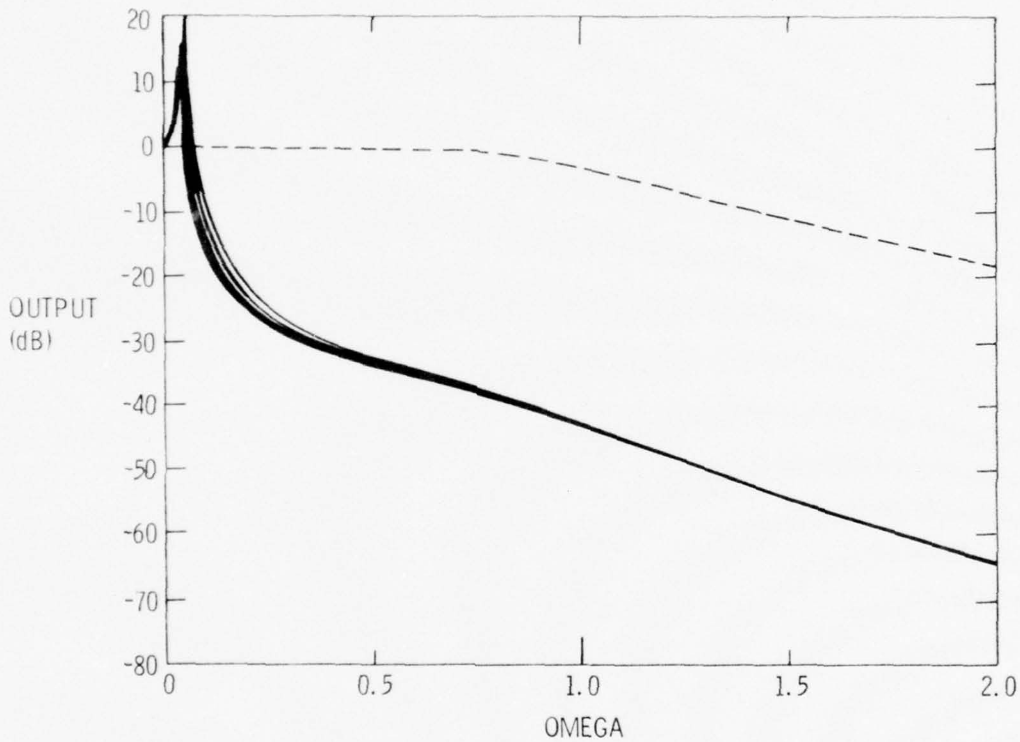


Figure 4.8(b) $|V_2/V_s|^2$ for Network in Figure 4.7

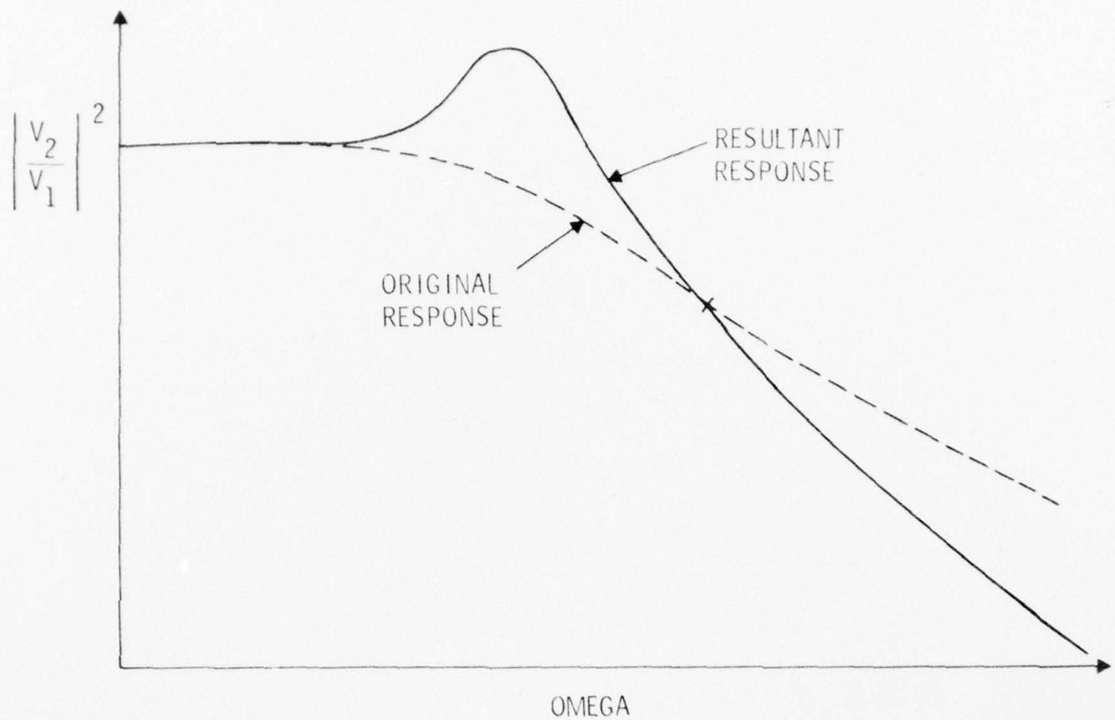


Figure 4.9 Raising Cutoff Frequency to Meet Response Criteria

4.4 Network Limitations

The networks synthesized by the Capacitor Shift method have a set of limitations similar to those of the Modified Transfer Admittance method. Specifically, there is the same upper limit on the stiffness of the final support spring and the method works well only for source masses in a particular range. The range, however, is different; the Capacitor Shift method working best for large source masses.

CHAPTER V

EXTENSIONS OF THE SYNTHESIS METHODS TO THE GENERAL CASE

5.1 General Limitations

In Chapter III, it was stated that although the purpose of this study was to establish a technique for the design of a filter with any general realizable transfer function, the initial study would be confined to a 3-pole Butterworth response. In this chapter, the results that were obtained will be summarized and extended to the general case.

In regard to the results obtained in Chapters III and IV, it must be remembered that a limitation of all electrical network synthesis techniques is that they must produce a reactive network terminated in a resistive load. It is not possible to specify what the response will be at any point within the reactive part of the network. All that may be specified is the network configuration. Thus, we must be satisfied with the element values as specified by the synthesis method. It is possible, however, to predict what the value of the final capacitor will be and its effect on the final network response. This will be discussed now in relation to the methods developed in Chapters III and IV.

5.2 Derivation of the Output Modifying Function for the M.T.A. Method

Recalling Figure 3.6 from Chapter III, it was shown that the response function of the network produced by the Modified Transfer Admittance method was different from that of the original desired

response function, i.e., the 3-pole Butterworth. The reason for the modified response is, of course, the series capacitor in the load. By modifying the original transfer admittance by the factor

$$\left|\frac{1}{1 + 1/as}\right|^2$$

and then multiplying the modified transfer admittance by

$$\left|\frac{(a + 2)^2}{a^2} + \frac{a + 2}{a^2 s}\right|^2$$

to find the quantity $|V_2/V_1|^2$, the original 3-pole Butterworth function has been changed by the factor [after normalization of H to $a/(a + 2)$]

$$\frac{1 + (a + 2)^2\omega^2}{1 + a^2\omega^2} , \tag{5.1}$$

where 'a' was the value of the original added capacitor. This output modifying function is plotted in Figures 5.1(a) and (b) showing, respectively, the overall curves and the low frequency detail for several functions of 'a'. As would be expected from Figure 3.6, the function rises quickly from zero dB at $\omega = 0$ to a constant for larger values of ω. The magnitude of the constant increase at high frequencies is inversely proportional to the value of 'a'.

By modifying Equation (5.1) to be [$H = a/(a+ L_s)$]

$$\frac{1 + (a + L_s)^2\omega^2}{1 + a^2\omega^2} , \tag{5.2}$$

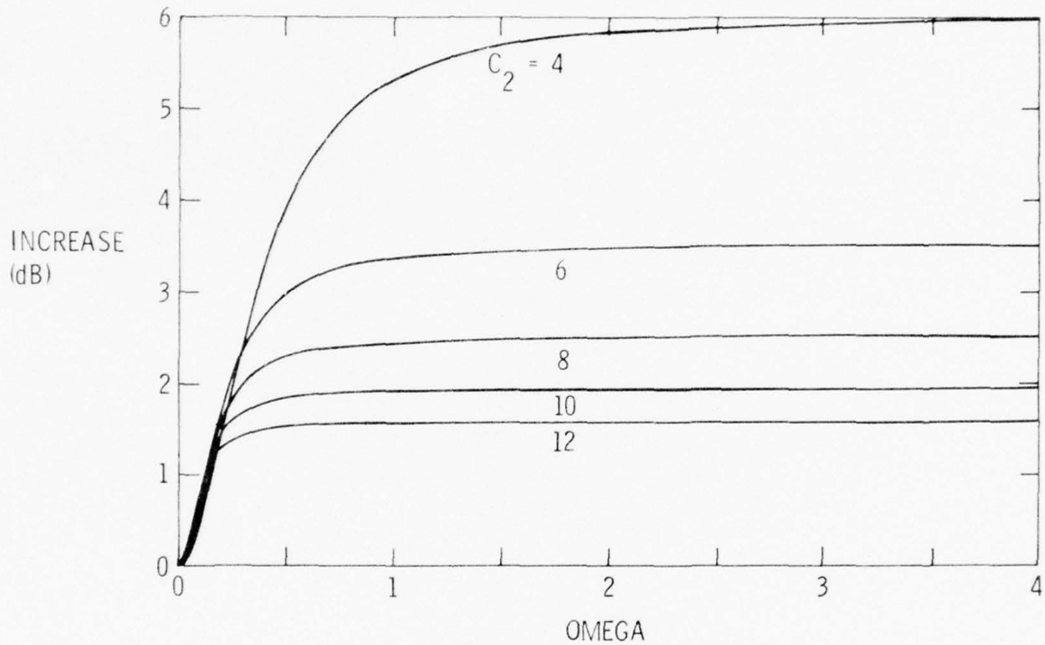


Figure 5.1(a) Output Modifying Function

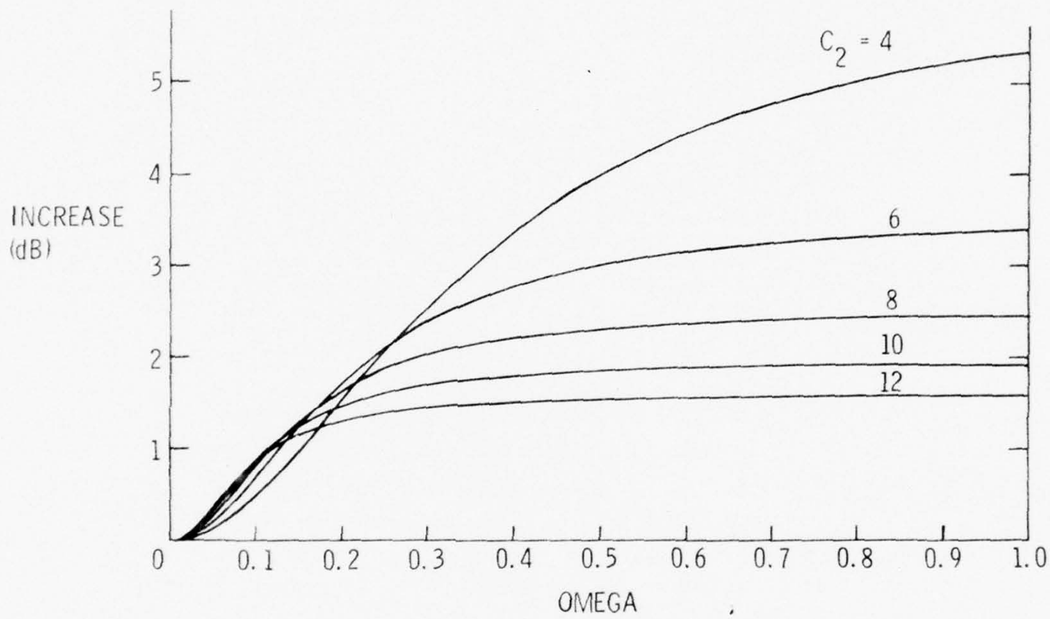


Figure 5.1(b) Output Modifying Function

where L_s is the total series inductance of the original network, Equation (5.2) will then be the correct modification function for <u>any network</u> synthesized by this method. The reason for using L_s will be made clear in Section 5.3. It is easy to see that Equation (5.2) can be obtained from Equation (5.1) by noting that the total series inductance of the original 3-pole Butterworth network (Figure 3.1) is $L_s = L_1 + L_2 = 2$.

5.3 Derivation of the Final Capacitor Lower Limit for the M.T.A. Method

The lower limit for the value of the final capacitor for any network synthesized by the Modified Transfer Admittance method is easily calculated. It is a function of the configuration and element values of the original network. Figures 5.2(a) and (b) show a general network which is being modified to give a series load capacitor using the Modified Transfer Admittance method. The original transfer admittance is defined as [for Figure 5.2(a)]

$$Y^T(s) = I_2/V_s \quad ,$$

and the modified transfer admittance is [Figure 5.2(b)]

$$Y_M^T(s) = \frac{as}{1 + as} \cdot Y^T(s) = I_3/V_s \quad ,$$

where 'a' is the value of the modifying capacitor. Since

$$Y^T(s) = \frac{Y_{12}(s)}{1 + Y_{22}(s)} = \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \ldots}{1 + a_1 s + a_2 s^2 + a_3 s^3 + \ldots} \quad ,$$

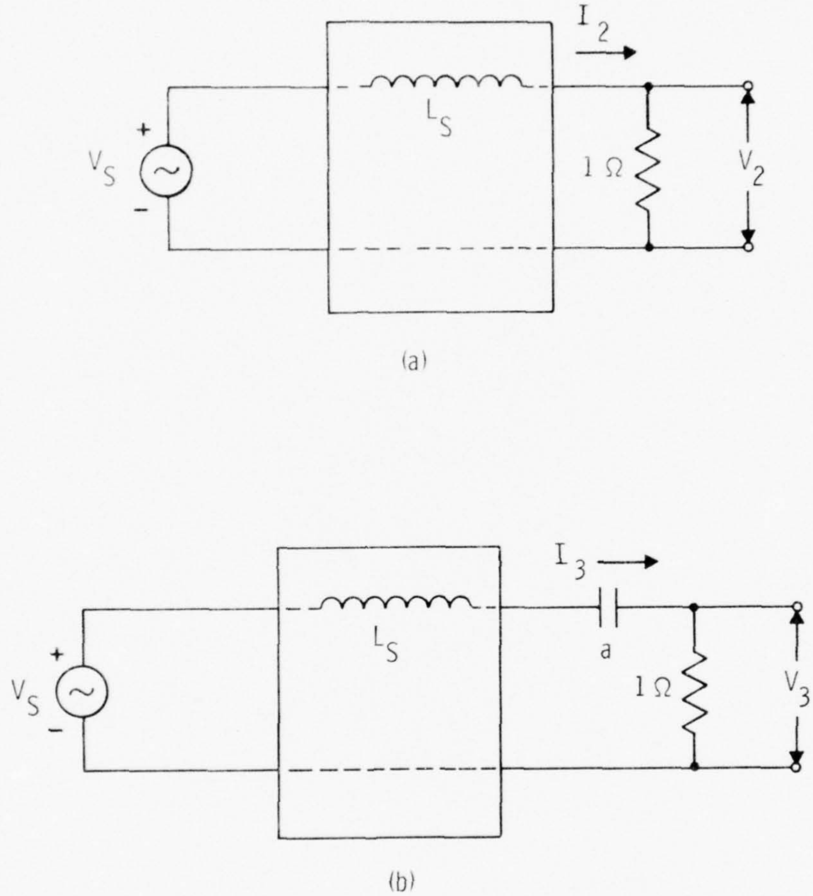


Figure 5.2 Modified Transfer Admittance Method on a General Impedance

then

$$Y_{22}(s) = \frac{a_1 s + a_3 s^3 + a_5 s^5 + \ldots}{1 + a_2 s^2 + a_4 s^4 + \ldots} ,$$

where the a_k and b_k are all positive. As the frequency becomes low,

$$\underset{s \ll 1}{\text{LIMIT}}\ Y_{22}(s) = a_1 s .$$

This has the form of an inductance and a_1 must therefore be the total series inductance L_s of the original network or $a_1 = L_s$.

The Modified Transfer Admittance method results in the network shown in Figure 5.2(b) where there is the required series capacitor; therefore,

$$Y_M^{T'}(s) = \frac{Y'_{22}(s)}{1 + Y'_{12}(s)} = \frac{as}{1 + as} \cdot Y^T(s)$$

$$= \frac{as(b_0 + b_2 s^2 + b_4 s^4 + \ldots)}{(1 + a_1 s + a_2 s^2 + \ldots) + as(1 + a_1 s + a_2 s^2 + \ldots)}$$

$$= \frac{as(b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \ldots)}{1 + (a_1 + a)s + (a_2 + aa_1)s^2 + (a_3 + aa_2)s^3 + \ldots} .$$

Then,

$$Y'_{22}(s) = \frac{1 + (a_2 + aa_1)s^2 + (a_4 + aa_3)s^3 + \ldots}{(a_1 + a)s + (a_3 + aa_2)s^3 + \ldots} .$$

For very low frequencies,

$$\underset{s \ll 1}{\text{LIMIT}}\ Y'_{22}(s) = 1/(a_1 + a)s$$

which has the form of a series capacitor. Since $a_1 = L_s$, the minimum value of the series capacitor is L_s, the total series inductance of the original circuit. This result will hold for any network having a Cauer I or similar (there may be resonances in the series arms) ladder structure.

5.4 Derivation of the Output Modifying Function for the C.S. Method

As with the Modified Transfer Admittance method, the Capacitor Shift method also produces a network with a response function that is different from the original desired response function. This difference is caused by the addition of an inductor across the source to produce a finite source mass and the requirement of a series capacitor in the load.

The modification function for the Capacitor Shift method is more complex than that of the Modified Transfer Admittance method. This function

$$[1 + 1/(a + L_s)^2\omega^2] \frac{M_0{}^2\omega^2}{|M_0 s + Z_{in}(s)|^2}$$

can be broken down into two parts. The first term is

$$|V_2/V'|^2 = [1 + 1/(a + L_s)^2\omega^2]$$

and relates the voltage across the resistor to the voltage across the resistor-capacitor combination. As in Section 5.2, L_s is the total series inductance of the original network. The proof that L_s is the correct value to use will be shown in Section 5.5.

The second term of the modification function contains the terms relating to the finite source mass. This term relates the current seen by the network with an infinite source mass to the current seen when the source is made finite:

$$|I_{in}/I|^2 = \frac{M_0^2\omega^2}{|M_0 s + Z'_{in}(s)|^2} .$$

The low frequency resonance is due to the term $Z'_{in}(s)$ in the denominator which is the input impedance of the network after the capacitor has been shifted and the transfer impedance has been normalized back to its original value.

5.5 <u>Derivation of the Final Capacitor Lower Limit for the C.S. Method</u>

The lower limit on the value of the load capacitor for the Capacitor Shift method may be easily calculated. Referring to Figure 5.3(a) and (b) and using the transform Equations (4.1a), (4.1b), and (4.1c), it is easily shown that

$$a' = \frac{a}{a + C_s}$$

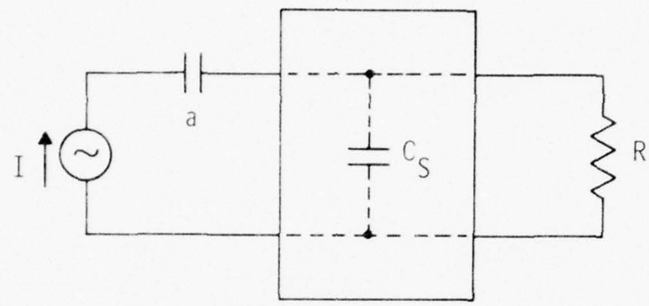
and

$$K = \frac{(a + C_s)^2}{a^2}$$

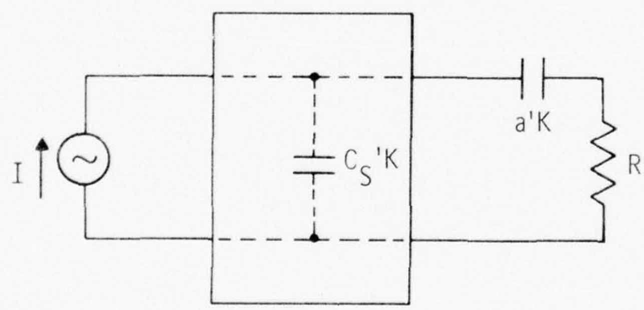
and that therefore (having normalized to the same load resistance),

$$a'K = a + C_s ,$$

where 'a' is the value of the inserted capacitor.



(a)



(b)

Figure 5.3 Shifting a Capacitor Through a General Network

This lower limit on the final capacitance (upper limit on the stiffness of the final supporting spring) raises many of the same problems as mentioned previously in Section 3.4 in regard to the Modified Transfer Admittance method. The solutions to these problems are the same; by raising the cutoff frequency, the final support spring stiffness is increased and masses internal to the filter are decreased. However, there is one major difference, that being that the source mass M_s is independent of the value of the final supporting spring. Because of this, the frequency of the resonant peak varies [Figures 4.5(a) and (b)] inversely with the value of the source mass. This, in effect, sets a lower limit on the value of the source mass, that limit being where the resonant peak moves into or near the rolled-off frequency band. The resonant peak moves lower in frequency with increasing source mass; hence, the statement that this method works best for larger source masses.

CHAPTER VI

THEORETICAL EXAMPLE

6.1 A Low Source Mass Force Generator

The following is an example of how the procedures previously presented may be applied to a practical situation. A machine weighing 51.5 Kg. has an out-of-balance mass rotating at 3600 rpm. This unbalanced mass produces a sinusoidal force at 60 Hz which is transmitted to the foundation. There are several other lower amplitude peaks at higher harmonics of the 60 Hz fundamental. The design objective is to reduce the force transmitted to the foundation by the vibrating machine. The fundamental must be reduced by at least 50 dB and the higher harmonics by at least 30 dB.

If the fundamental were specified to occur at twice the cutoff frequency, there would be several ways of obtaining the design specifications. The first would be to use an 8-pole Butterworth filter which would be down approximately 48 dB at twice the cutoff frequency. This network would be rather complex and, since the higher harmonics need not be reduced so severely, a simpler alternative is to use a 4-pole Butterworth filter with a zero at twice the cutoff frequency. This allows the fundamental to be well suppressed without so severely reducing the higher harmonics.

Since the source mass is small, the Modified Transfer Admittance method will be used. The transfer admittance describing the 4-pole Butterworth with a zero at twice the cutoff frequency is (for a cutoff frequency of 1 radian/second)

$$|Y^T(s)|^2 = \frac{(1 + s^2/4)^2 \; H^2}{1 + s^8} .$$

The modified transfer admittance is

$$|Y_M^T(s)|^2 = \frac{(1 + s^2/4)^2 \; H^2}{1 + s^8} \cdot \left|\frac{a \cdot s}{1 + a \cdot s}\right|^2$$

and the resulting electrical and mechanical networks are shown in Figures 6.1(a) and (b), respectively. The approximate element values are as follows:

$$M_S = \frac{1211a^8 + 12660a^7 + 54512a^6 + 12334a^5 + 182547a^4}{644a^8 + 8920a^7 + 51018a^6 + 155025a^5 + 268287a^4}$$

$$\frac{+ 172649a^3 + 106339a^2 + 36384a + 4372}{+ 265300a^3 + 147161a^2 + 45905a + 5257} ,$$

$$S_1 = \frac{1211a^8 + 12660a^7 + 54512a^6 + 12334a^5 + 182547a^4}{1870a^8 + 22652a^7 + 111384a^6 + 287155a^5 + 421634a^4}$$

$$\frac{+ 172649a^3 + 106339a^2 + 36384a + 4372}{+ 368547a^3 + 200484a^2 + 63665a + 9946} ,$$

$$M_2 = \frac{246a^4 + 128a^3 + 1938a^2 + 260a + 69}{4(83a^4 + 493a^3 + 913a^2 + 561a + 71)}$$

$$S_2 = 4M_2$$

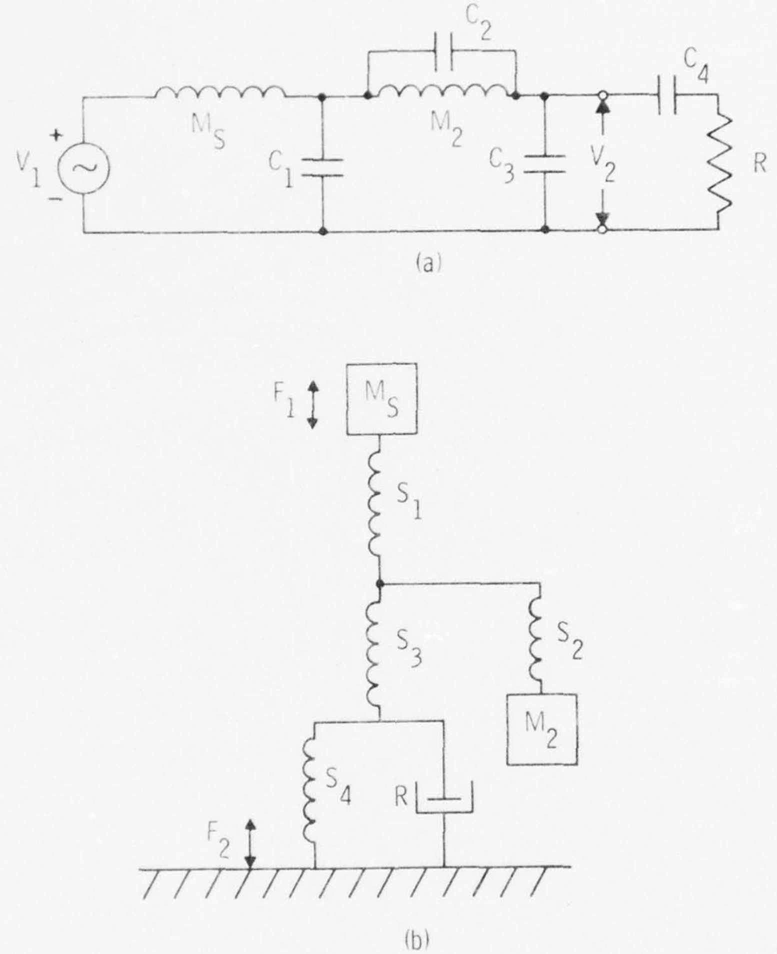


Figure 6.1 Analagous 4-Pole Butterworth Networks with a Zero at Twice the Cutoff Frequency

$$S_3 = \frac{31a^2 + 82a + 17}{3.5a^2 + 0.9a - 20.5}$$

and

$$S_4 = \frac{1}{a + \sqrt{4 + 2\sqrt{2}}} .$$

The element values for three different values of 'a' are shown in Figure 6.2.

Since spring S_3 has twice the stiffness at $a = 3$ than at $a = 5$, upon normalization to the correct cutoff frequency, its stiffness might become prohibitively large. Therefore, $a = 5$ will be tried first. When normalized to a cutoff frequency of 30 Hz, the element values are:

$$M_s = 0.0054 \text{ Kg} ,$$
$$S_1 = 86 \text{ N/m} ,$$
$$M_2 = 0.00209 \text{ Kg} ,$$
$$S_2 = 296 \text{ N/m} ,$$
$$S_3 = 3170 \text{ N/m} ,$$
$$S_4 = 24.7 \text{ N/m}$$

and

$$R = 1 \text{ Mechanical Ohm} .$$

It can be seen that increasing the element values by a factor of 10,000 will bring the required source mass near the value of the mass of the machine. Therefore, the final element values are:

	a = 10	a = 5	a = 3
M_s	1.37 Kg	1.02 Kg	.76 Kg
S_1	0.547 N/m	0.456 N/m	0.382 N/m
M_2	0.49 Kg	0.394 Kg	0.60 Kg
S_2	1.96 N/m	1.57 N/m	2.42 Kg
S_3	11.63 N/m	16.81 N/m	39.56 N/m
S_4	0.079 N/m	0.131 N/m	0.178 N/m
R	1 Mech. Ω	1 Mech. Ω	1 Mech. Ω

Figure 6.2 Table of Element Values for Figure 6.1 as a Function of 'a'.

$$M_s = 54 \text{ Kg} ,$$

$$S_1 = 8.6 \times 10^5 \text{ N/m} ,$$

$$M_2 = 20.6 \text{ Kg} ,$$

$$S_2 = 2.96 \times 10^6 \text{ N/m} ,$$

$$S_3 = 3.17 \times 10^7 \text{ N/m} ,$$

$$S_4 = 2.47 \times 10^5 \text{ N/m}$$

and

$$R = 10{,}000 \text{ Mechanical Ohms} .$$

The original machine needs to have 3.5 Kg additional mass added to it in order to satisfy the network requirements. The alternative would be to normalize to a resistance of 9,337 Mechanical Ohms. In this case, no additional mass need be added.

The output modifying function for this system is

$$\frac{1 + (a + \sqrt{4 + 2\sqrt{2}})^2 \omega^2}{1 + a^2 \omega^2}$$

$[H = a/(a + \sqrt{4 + 2\sqrt{2}})]$ and is shown for several values of 'a' in Figure 6.3. Figure 6.4 shows the response $|V_2/V_1|^2$ of the network in Figure 6.1(a) for $a = 5$. As expected, the extra capacitor has not changed any of the characteristics of the zero in the response except that it is a little narrower. This narrowing of the zero means that the tolerance of the element values must be smaller in order for the frequency of the zero to exactly match that of the fundamental.

While the subject of internal damping has not been widely discussed in this paper, it is easy to show the effects of damping in

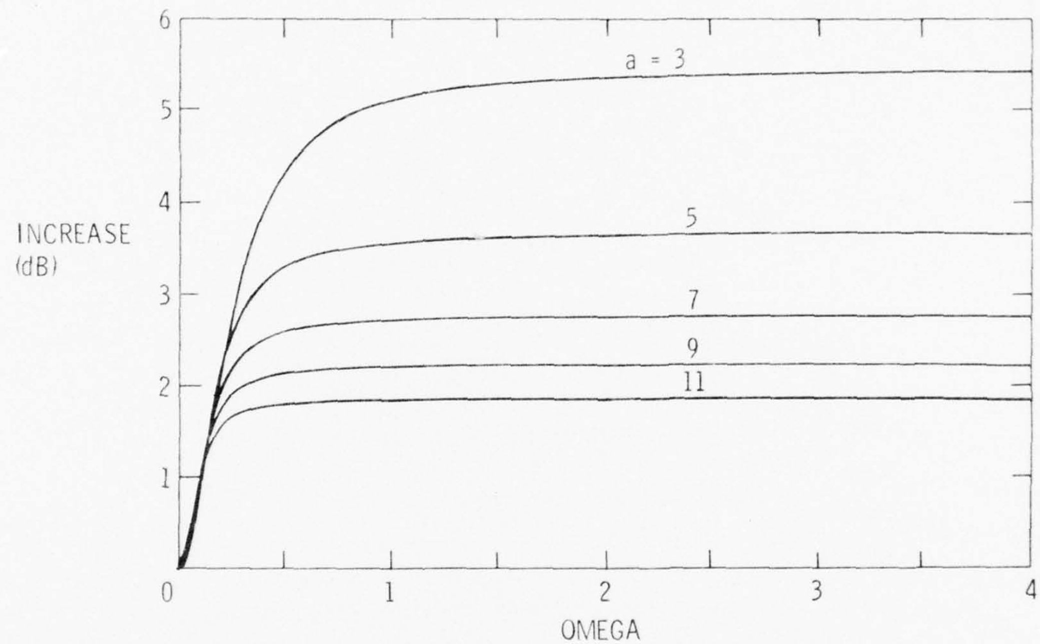


Figure 6.3 Output Modifying Function for 4-Pole Butterworth Network

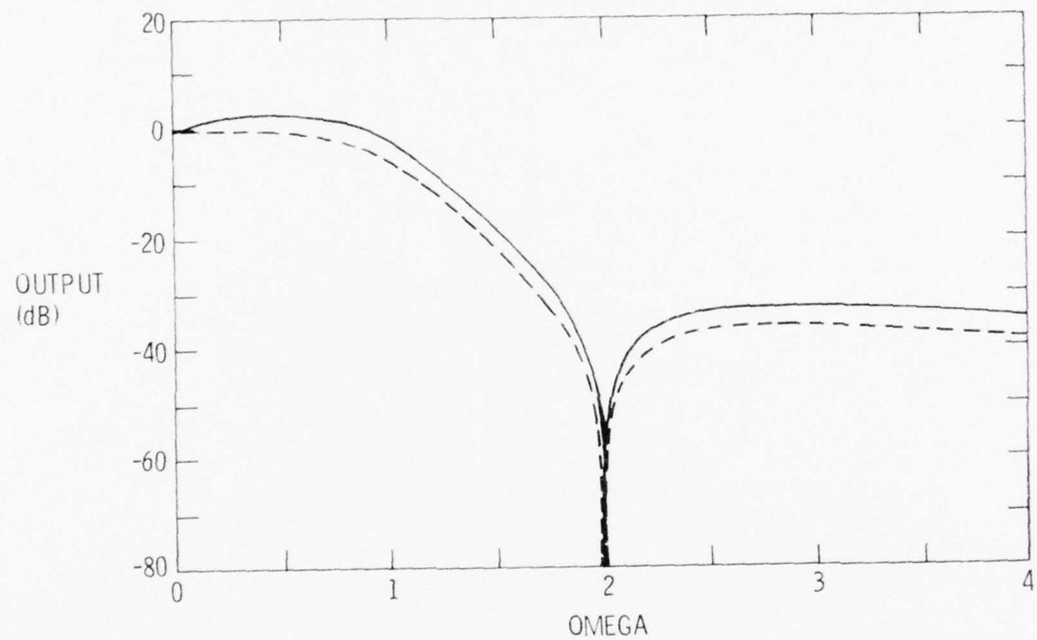


Figure 6.4 $|V_2/V_1|^2$ for the Network in Figure 6.1(a)

the resonant circuit consisting of elements S_2 and M_2. Figure 6.5(a) shows the configuration of a damped resonant circuit. A damped spring, however, is equivalent to a capacitor in series with a resistor - not in parallel. Therefore, the resonant circuit consisting of S_2 and M_2 has the elements in Figure 6.5(b) as its electrical equivalent. The elements C' and R' are slightly different from C and R at resonance. Since a mass cannot be lossy, the loss associated with it can be lumped in with the loss of the spring, therefore achieving the same overall damping. The damping in the spring is $R + R'$ or approximately $2R$. Figures 6.6(a) through (d) show the network in Figure 6.1(a) when the spring S_2 is lossy. The overall damping varies from 0.0001 to 0.1, respectively, or twice that amount in the spring alone.

There obviously must be a limit to the amount of damping allowable in the resonance arm. Too much damping will not allow the design objective of a 50 dB reduction in the force of the fundamental to be met. This is clearly seen in Figure 29(d). The highest allowable overall damping is about 0.01 as shown in Figure 6.6(c). Figure 6.7 shows a detail of the response near the zero. The response of the modified network is down 60 dB at twice the cutoff frequency, thus allowing approximately a ±3% tolerance in frequency at 50 dB down. Notice that, above the resonance, there is no longer a 4-pole response. This is because the spring S_2 shunts the mass M_2 at high frequencies. The resulting response is that of a 2-pole filter with a 12 dB per octave drop above the resonance. This is clearly shown in Figure 6.8.

All of the requirements of the filter have therefore been met with this design. The resonance reduces the fundamental by at least 50 dB for damping less than 0.01, and the higher harmonics have been reduced by over 30 dB. The element values are not unrealistic and should be easily obtainable.

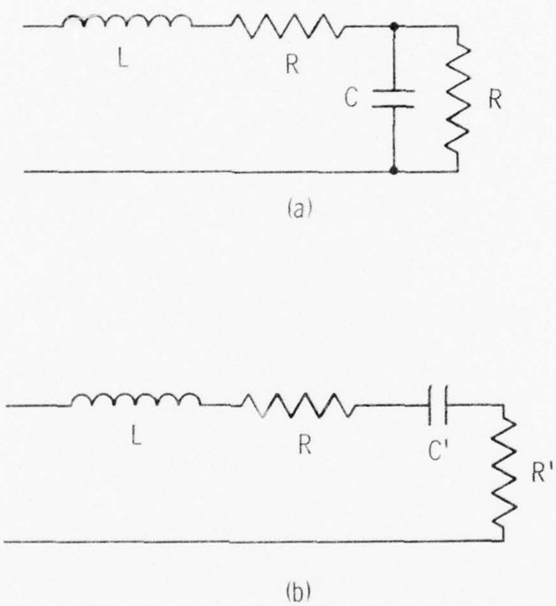


Figure 6.5 Equivalent Damped Resonant Circuits

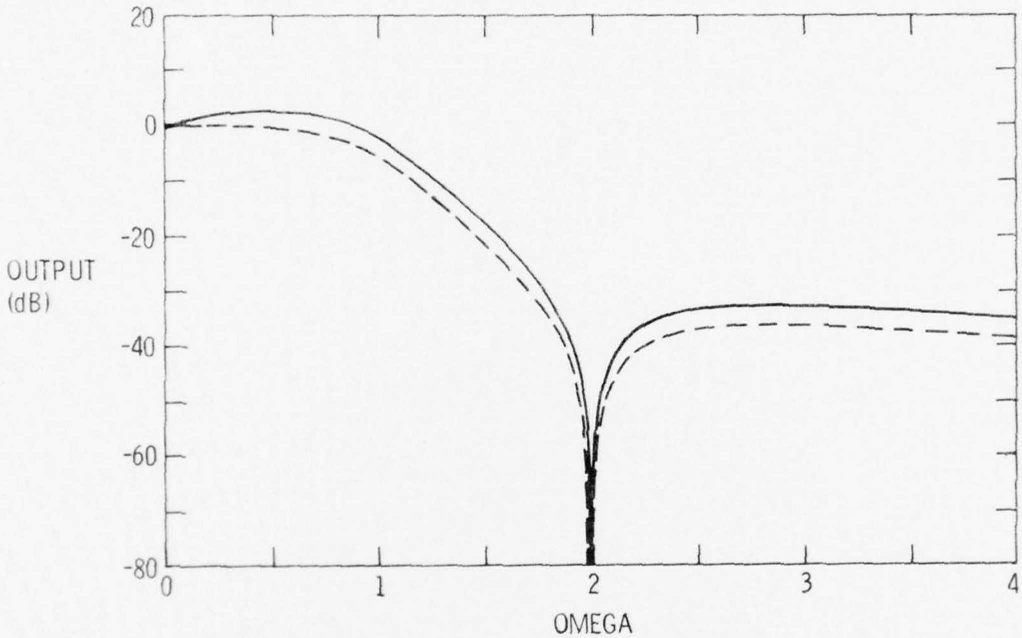


Figure 6.6(a) $|V_2/V_1|^2$ for the Network in Figure 6.1(a) with Damping in the Resonant Arm

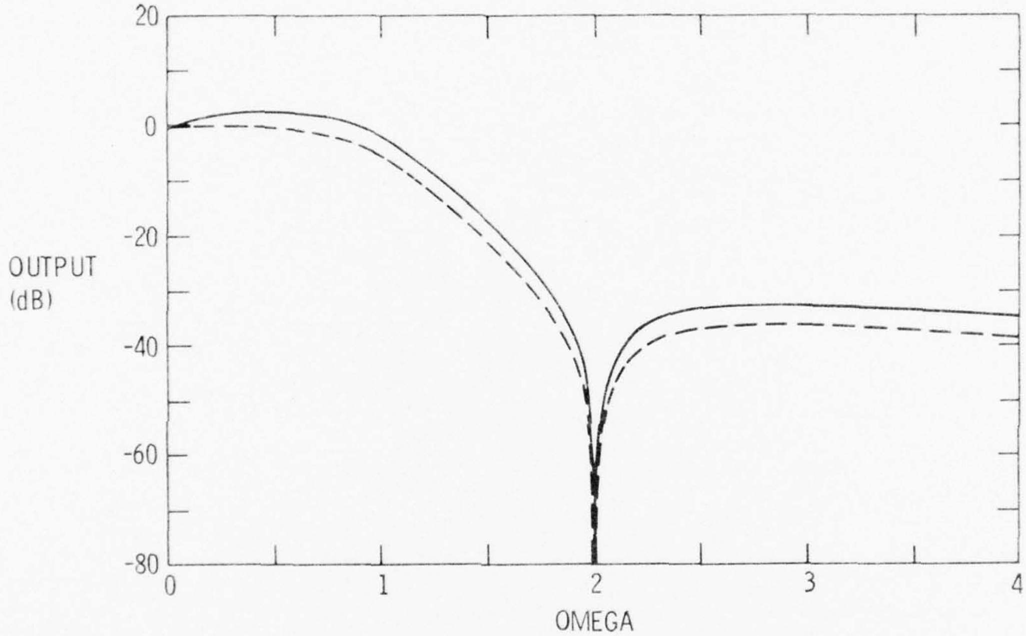


Figure 6.6(b) $|V_2/V_1|^2$ for the Network in Figure 6.1(a) with Damping in the Resonant Arm

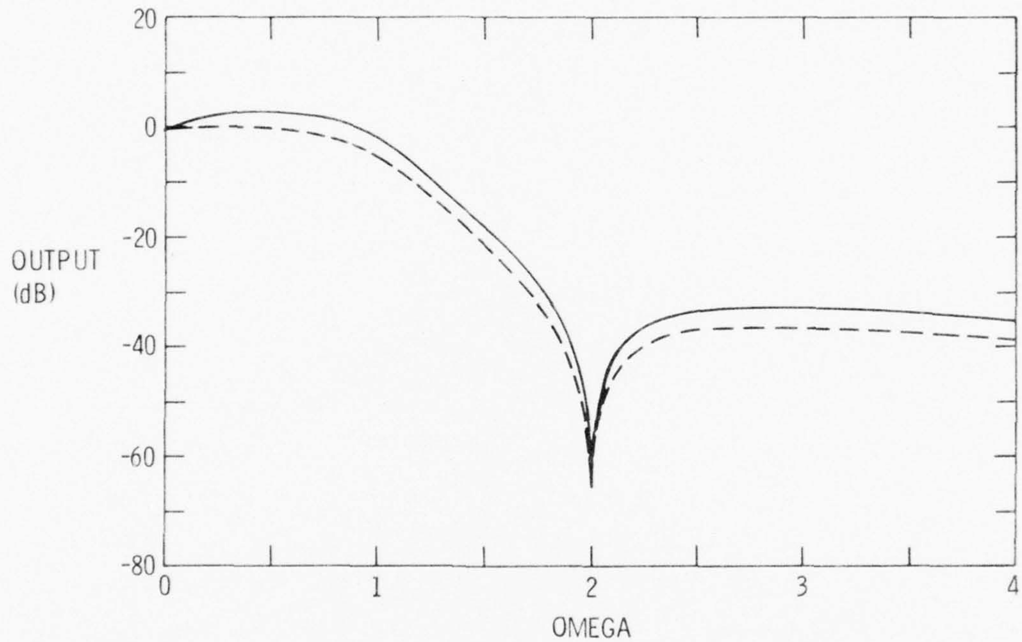


Figure 6.6(c) $|V_2/V_1|^2$ for the Network in Figure 6.1(a) with Damping in the Resonant Arm

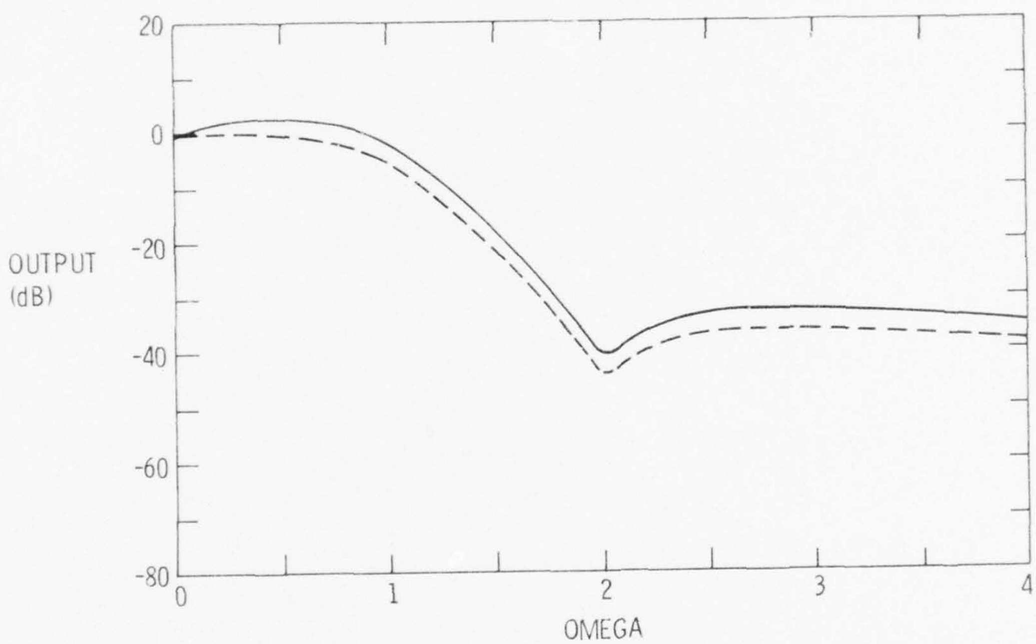


Figure 6.6(d) $|V_2/V_1|^2$ for the Network in Figure 6.1(a) with Damping in the Resonant Arm

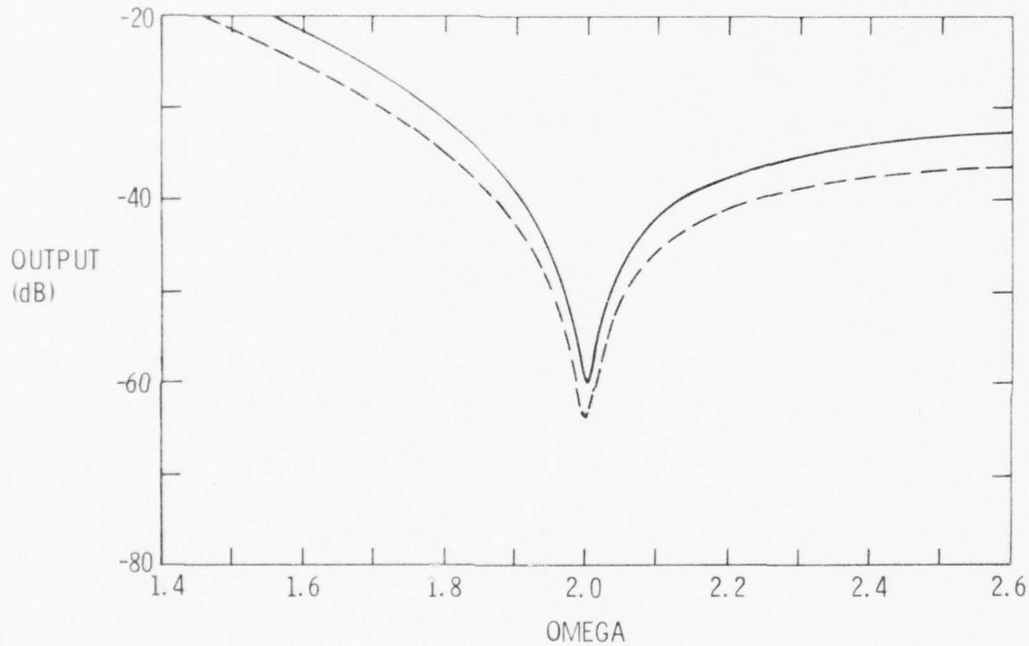


Figure 6.7 Detail of Response Near Resonance

AD-A034 460 PENNSYLVANIA STATE UNIV UNIVERSITY PARK APPLIED RESE--ETC F/G 13/9
MECHANICAL ANTIVIBRATION FILTER DESIGN USING ELECTRICAL NETWORK--ETC(U)
APR 76 C B PUTMAN
UNCLASSIFIED TM-76-162 N00017-73-C-1418
NL

2 OF 2
ADA034460

END
DATE
FILMED
2 - 77

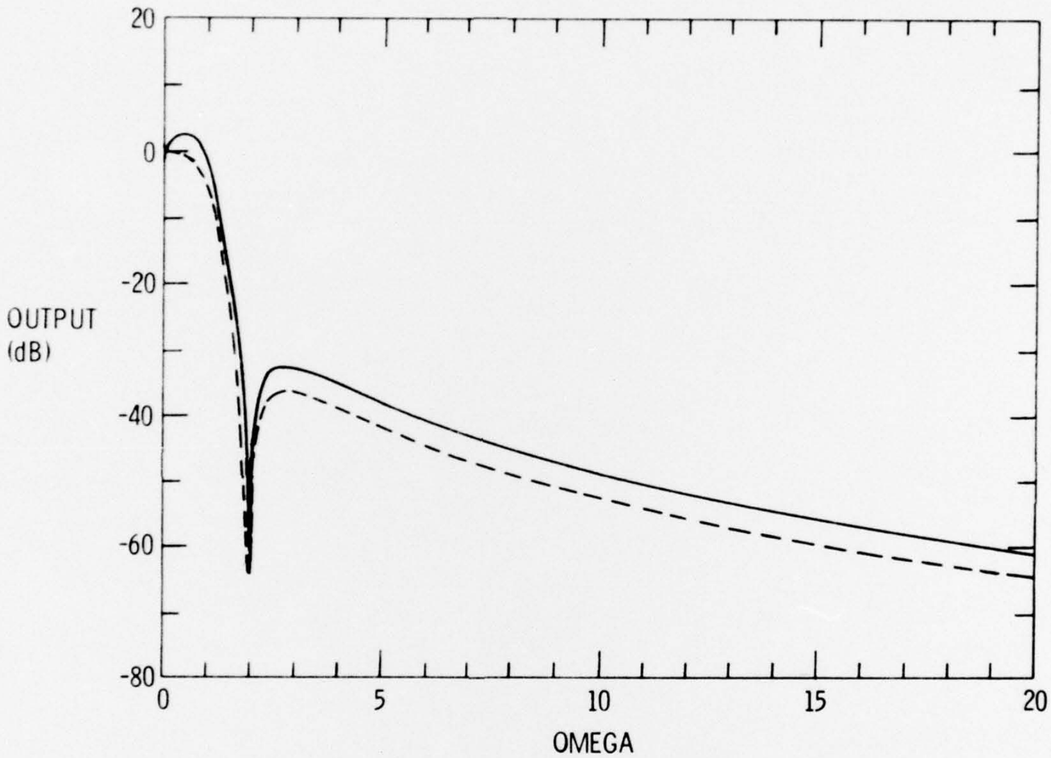


Figure 6.8 $|V_2/V_1|^2$ for the Network in Figure 6.1(a) Showing 12 dB Per Octave Rolloff Above Resonance

CHAPTER VII

CONCLUSION AND RECOMMENDATION FOR FURTHER RESEARCH

It has been shown in Chapters III and IV that it is possible to use electrical network synthesis techniques to design mechanical anti-vibration filters of any degree of complexity. The methods and network designs produced by these methods meet all four of the criteria as outlined in the beginning of Chapter II. This chapter will summarize the strengths and weaknesses of the two methods along with proposals for areas for further research.

The Modified Transfer Admittance method produces a network driven by a constant force generator in which the source mass is an integral part of the low-pass filter. The transmissibility of this type of network is related to the original proposed transfer admittance by the modification function

$$\frac{1 + (a + L_s)^2 \omega^2}{1 + a^2\omega^2} ,$$

where L_s is the total series inductance of the network having the proposed transfer admittance and 'a' is the value of the modifying capacitor. This modification function causes the transmissibility of the low-pass filter to rise at low frequencies (relative to the desired response) and then to remain at a constant difference from the desired

response at high frequencies. The magnitude of this difference depends on the value of the inserted capacitor. This value, while arbitrary, does have a lower limit, thus restricting the maximum stiffness of the support spring which can cause some problems.

The Capacitor Shift method produces a network driven by a constant velocity source. This method favors high source masses and produces a configuration where the source mass is not part of the filter. A large resonant peak is produced at low frequencies and the transmissibility drops 6 dB per octave more rapidly than the transfer impedance. There is also quite a large offset in the transmissibility function which is proportional to the log of the inverse of the mass. This offset causes the filter to be able to meet its criteria at a lower cutoff frequency than the Modified Transfer Admittance method. The transfer impedance of this network has the same low frequency resonant peak but, above the cutoff frequency, has the proposed response function. The Capacitor Shift method also has the restriction of a maximum support spring stiffness. However, this restriction is not so serious in this case. Since the source mass is independent of the filter, the filter impedance can be changed without neccessitating a change in the impedance of the source mass. The peaks in the transmissibility function do not present a serious problem as techniques have been developed for reducing or eliminating them by the use of dynamic absorbers.[2,7,8]

A potential source of trouble in both methods is the possibility of abnormally large or small element values. Upon synthesizing filter networks, it is not possible to specify what the element values will be. Occassionally, the synthesis will require an element magnitude to be

quite a bit larger or smaller than the other elements in the network. This can cause problems upon changing the overall impedance or cutoff frequency of the network in that the magnitude of the component can exceed the bounds for practical implementation.

The methods presented in this paper were derived for perfect, massless and lossless springs since this represented a first attempt at formulating a general design theory. Any real spring, of course, cannot meet these requirements and results must differ to some degree from those presented in this paper. A possible area for further study would be to define the limits for which the results of this study apply. Other possible areas of further study include the adjustment of abnormal element values and research into synthesis methods that allow for lossy, non-linear or non-lumped components.

BIBLIOGRAPHY

1. Darlington, S., "Synthesis of Reactance 4-Poles which Produce Prescribed Insertion Loss Characteristics," J. Math. and Physics, 18(4), 257, (1939).

2. Snowdon, J. C., Vibration and Shock in Damped Mechanical Systems, John Wiley and Sons, (1968).

3. Storer, J. E., Passive Network Synthesis, McGraw-Hill Electrical and Electronic Engineering Series, 158-160, (1957).

4. Cauer, W., Die Verwicklichung Von Wechselstromwider Ständen Vorgeschriebener Frequenzabhängigkeit, Arch. Elektrotech., 17, 355, (1927).

5. Cauer, W., "Filters Open Circuited on the Output Side," Elek. Nachr.-Tech., 16(6), 161-163, (1939).

6. Guillemin, E. A., "A Summary of Modern Methods of Network Synthesis," Adv. Elect., Vol. 3, 261, (1951).

7. Snowdon, J. C., "Steady State Behavior of the Dynamic Absorber," J. Acoust. Soc. Am., 31, 1668-1675, (1959).

8. Ormondroyd, J., and Den Hartog, J. P., "The Theory of the Dynamic Vibration Absorber," Trans. ASME, 50, A9-A22, (1928).

DISTRIBUTION

Commander (NSEA 09G32) Naval Sea Systems Command Department of the Navy Washington, D.C. 20362	Copies 1 and 2
Commander (NSEA 0342) Naval Sea Systems Command Department of the Navy Washington, D.C. 20362	Copies 3 and 4
Defense Documentation Center 5010 Duke Street Cameron Station Alexandria, VA 22314	Copies 5 through 16